Name: ____

HW 6: Chapter 6

1. Escape from Planet X

Astronomers believe that a large, unseen planet - Planet X, also known as Planet Nine - may be lurking in the distant reaches of our solar system. Its gravitational influence could explain the strange, clustered orbits of icy bodies far beyond Neptune. Though it hasn't been directly observed, its presence is strongly suspected. Your spacecraft has landed on Planet X as part of a bold exploratory mission. Based on gravitational measurements, Planet X is estimated to have a mass that is 5x that of Earth of $M_E = 6.0 \times 10^{24}\ kg$ and a radius that is 2x that of Earth of $R_E = 6.0 \times 10^6\ m$. With your mission complete, it's time to leave. To return home, your rocket must escape Planet X's gravity – that is, reach a point infinitely far away with zero speed. There is just one problem – you only have enough fuel for one good rocket thrust.

Why You Can't Use $PE = mgh$: On Earth, we often estimate gravitational potential energy as $PE = mgh$. But that formula assumes gravity stays constant over the height you're lifting something – it only works near the surface of a planet. To escape a planet entirely, you're traveling far enough that gravity gets noticeably weaker. In this case, you need the far-field gravitational potential energy:

$$PE = -\frac{GMm}{r}$$

This formula works at any distance, near or far, is zero only at infinity, and is negative near the planet, meaning that you're in a gravitational well. To escape, the rocket must gain enough energy to climb out of that well (Hint: you want 0 left over velocity at a distance of ∞)

Your task: Use the work-energy theorem to calculate the minimum speed the rocket must have at the surface of Planet X to escape its gravity.

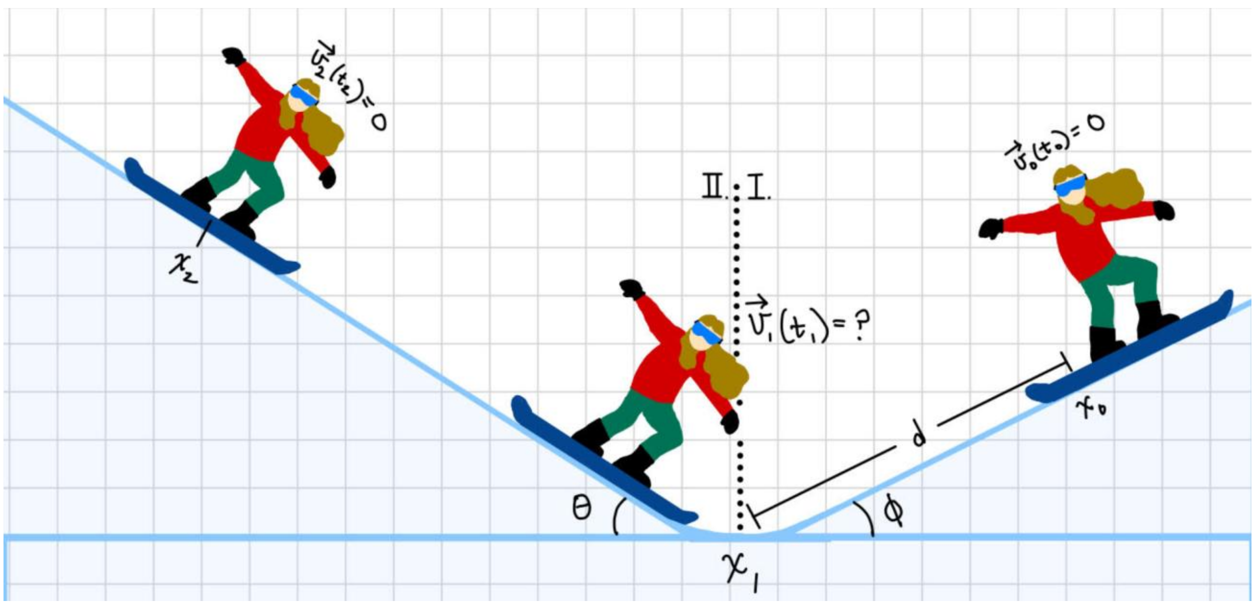
- The rocket starts at rest on the surface and receives a single instantaneous push (no further thrust)
- There is no atmosphere or friction.
- Only gravity acts after launch.

2. Return of the Snowboarder

A snowboarder, with a mass of 54kg, starting from rest, slides down a snow-covered incline that is 30^0 from the horizontal. After sliding down a distance $d = 20m$ she immediately ascends another incline angled $45^0$from the horizontal. The coefficient of kinetic friction is 0.03. Neglect air resistance. Use the work-energy theorem for this problem.

I. What is her final velocity coming off the ramp?

II. Assume that the final velocity that you calculated in part I is her initial velocity as she begins her ascent up the incline. How far up the ramp does she go?



3. The Mars Rover – not as hard as you'd think!

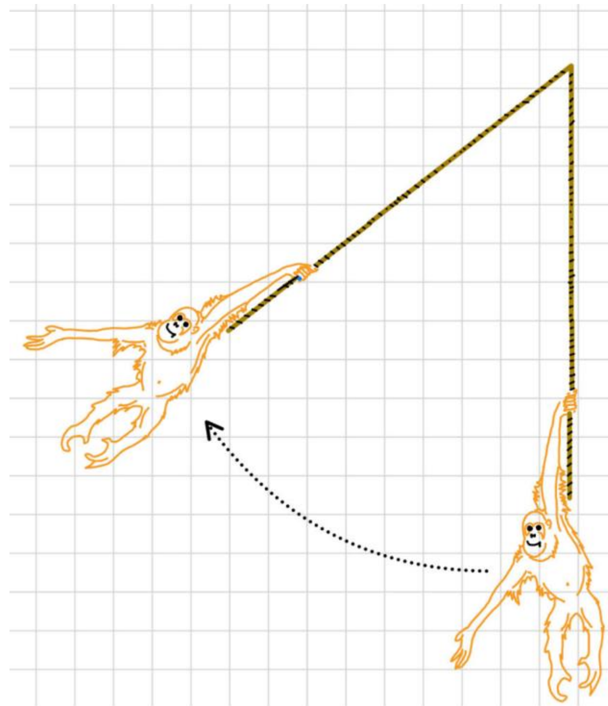
A Mars rover is perched at the top of a smooth, frictionless hill. It rolls down the slope, through a valley, and up a second hill of the same shape and height. The rover's engine is never turned on during the trip. What is the total work done on the rover by all forces over the entire trip from start to finish?

Solution:

The hint really is in the title - this one's simpler than it looks. Since the rover never used its engine and there's no friction or air resistance, only conservative forces are acting. The second hill is identical to the first, so the rover ends at the same height it started from. That means it must have the same speed, zero, at the top. In the valley, it would coast at constant speed because no net force acts on it. Since it returns to its original energy state, the net work done on the rover is zero:

4. How High Will Gunjito Go?

The Amazing Gunjito, $m = 75\ kg,$ swings down from a tree using a rope that is $l = 7\ m$ long and at the bottom of his swing he has a maximum velocity of $v = 4\ m/s$. Neglect air resistance.



a. Where in his swing is his kinetic energy maximum?

b. Where in his swing will his potential energy be minimum?

c. What is the maximum height and angle that Gunjito will reach?

d. How much work is done by gravity to bring Gunjito to his maximum final height from the height he originally swung down from?