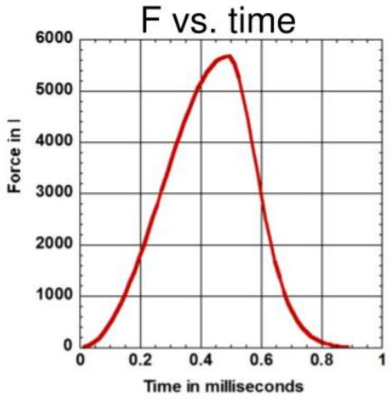
Impulse and Momentum

Chapter 7

Time Varying Force

- In the video, a baseball is hit by a bat and captured 8,800 fps.
- Just before the bat touches the ball, $t_0, F = 0$
- During contact, force rises to a maximum and then decreases back to 0.
- Over $\Delta t = t_f - t_0, F$ changed. $F = F(t)$.

- Our current tools do not allow us to describe how a time-varying force affects the motion of an object.
- Two ideas will help us do just this:
  - Impulse
  - Linear Momentum

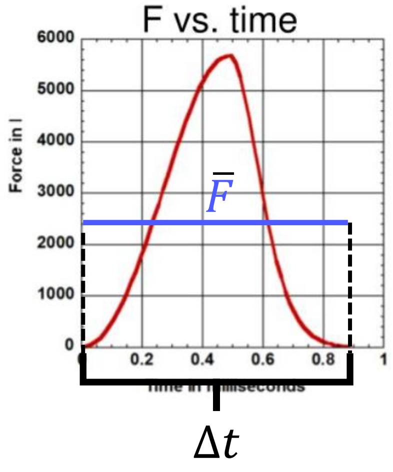


Impulse and Momentum Defined

Definition of Impulse

The impulse $\vec{J}$ of a force is the product of the time averaged force $\vec{\bar{F}}$ and the time interval Δt during which the force acts:

$$\vec{J} = \vec{\bar{F}}\Delta t$$



Impulse is a vector quantity and has the same direction as the time averaged force.

SI Unit of Impulse: $N \cdot s$

Definition of Linear Momentum

The linear momentum $\vec{p}$ of an object is the product of the object's mass m and velocity $\vec{v}$:

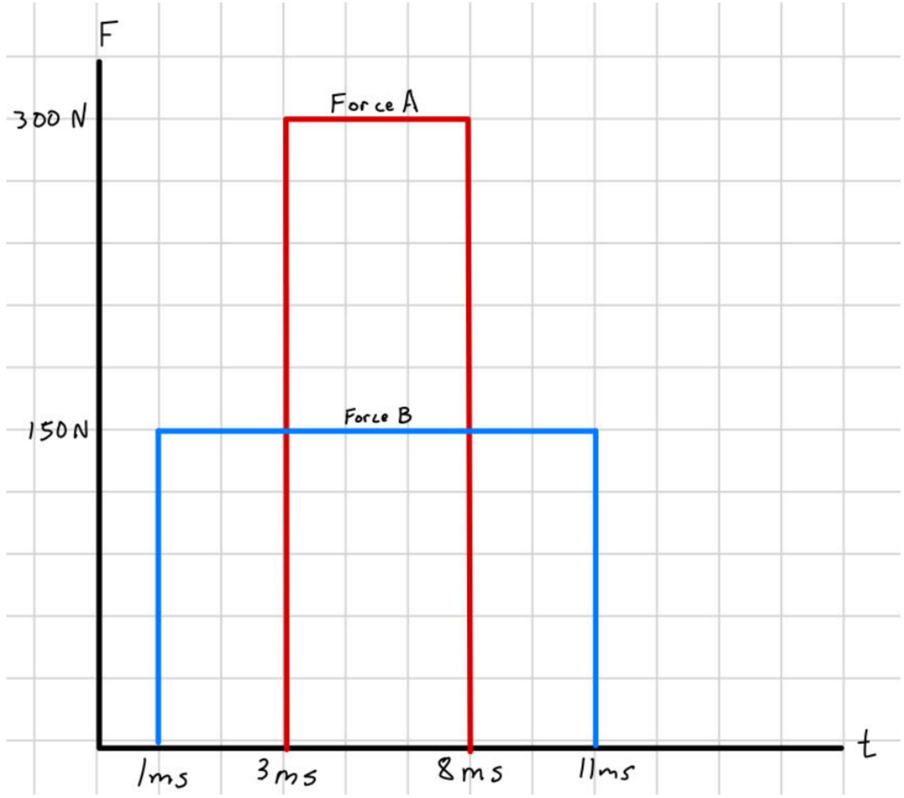
$$\vec{p} = m\vec{v}$$

Linear momentum is a vector quantity that points in the same direction as the velocity.

SI Unit of Linear Momentum: $kg \cdot m/s$

Team Activity: Concept Question 7.1

The two graphs time averaged force-versus-time data for two collisions. Which force delivers the greater impulse?



Impulse-Momentum Theorem

- We can use Newton's 2nd Law to reveal a relationship between impulse and linear momentum,

$$\sum\vec{\bar{F}} = m\vec{a} = m\left(\frac{\vec{v}_f - \vec{v}_0}{\Delta t}\right) = \frac{m\vec{v}_f - m\vec{v}_0}{\Delta t}$$

$$\vec{J} = \sum\vec{\bar{F}}\Delta t = m\vec{v}_f - m\vec{v}_0 = \vec{p}_f - \vec{p}_0$$

- Amazing! Why? During a collision, it is difficult to determine $\sum\vec{\bar{F}}$ but it is easy to measure the velocities!

Impulse-Momentum Theorem

When a net average force $\sum\vec{\bar{F}}$ acts on an object during a time interval Δt, the impulse of this force is equal to the change in momentum of the object:

$$\vec{J} = m(\vec{v} - \vec{v}_0) = \Delta p$$

The effect of an impulsive force is to change the object's momentum from $\vec{p}_0$ *to* $\vec{p}_f = \vec{p}_0 + \vec{J}$,.

Example: Calculating the change in momentum

A ball of mass $m = 0.25\ kg$ rolling to the right at $1.3\ m/s$ strikes a wall and rebounds to the left at $1.1\ m/s$. (a). What is the change in the ball's momentum? (b). What is the impulse delivered to it by the wall?

Solution: (a).The x-component of the initial momentum is,

$$p_{x,0} = mv_{x,0}$$

The x-component of the final momentum is,

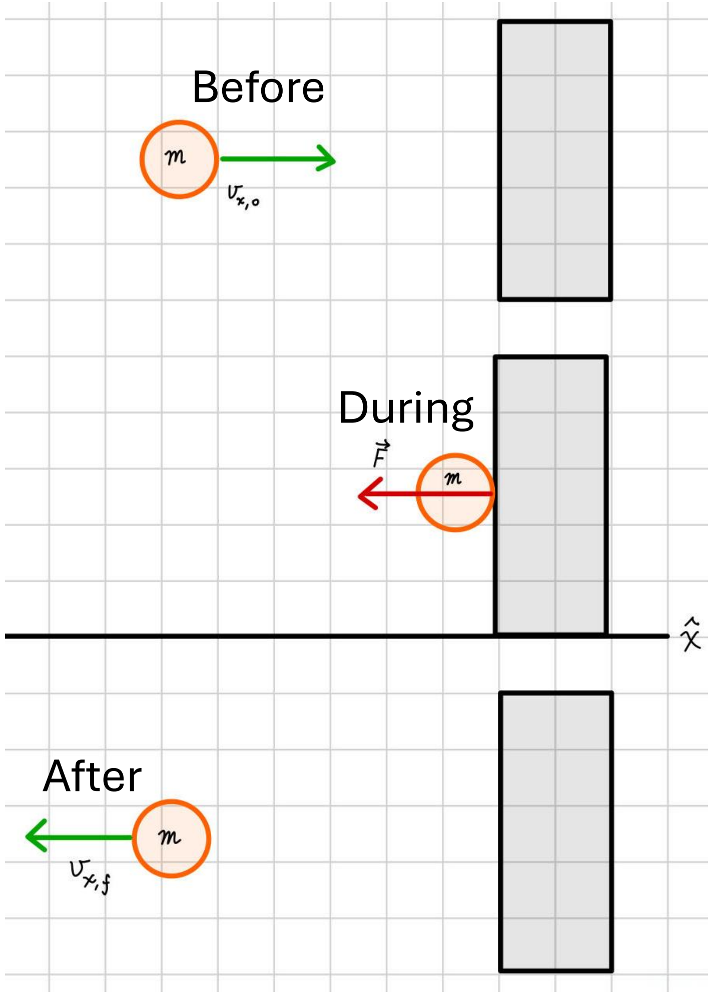
$$p_{x,f} = -mv_{x,f}$$

The change in the x-component of the momentum,

$$\Delta p_x = p_{x,f} - p_{x,0} = m\left(-v_{x,f} - v_{x,0}\right) = 0.25(-1.3 - 1.1)kg \cdot m/s$$
$$= -0.60\ kg \cdot m/s$$

(b). Now use the impulse-momentum theorem,

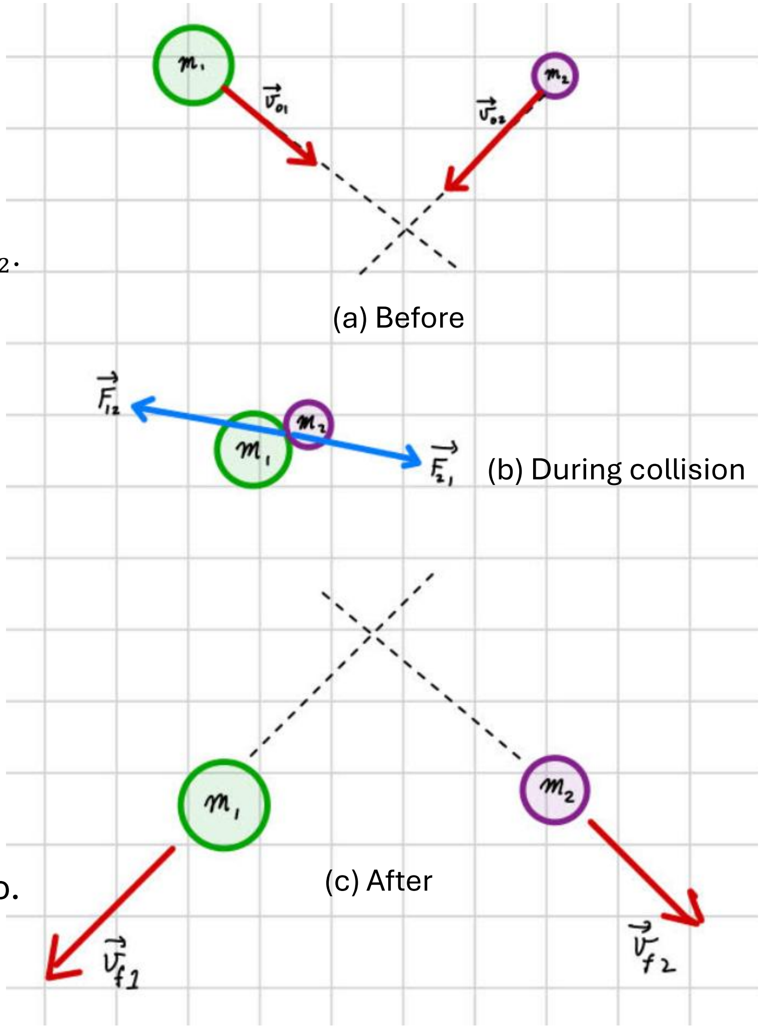
$$J_x = \Delta p_x = -0.60\ kg \cdot m/s$$



When the ball hits the wall, it deforms, creating internal motion and losing energy. The impulse–momentum theorem still gives the impulse for translational motion, but the total impulse from the force–time integral also includes the part that went into deformation and internal motion. In a perfectly elastic collision, the change-in-momentum method and the force–time method give the same impulse. In an inelastic collision, they differ.

The Principle of Conservation of Linear Momentum

- The system consists of two objects in motion.
  - a. Collision course: the objects are moving with initial velocities toward each other.
  - b. The objects collide
  - c. The objects depart with final velocities.
- Two types of forces act on the system:
  1. Internal forces: forces objects within the system exert on each other. Examples: $\vec{F}_{12}$ and $\vec{F}_{21}$.
  2. External forces: forces exerted on the objects by agents external to the system. Examples: $\vec{F}_{g1}$ and $\vec{F}_{g2}$.
- Use the Impulse-Momentum Theorem:
  - Object 1: $\left(\vec{F}_{g1} + \vec{\bar{F}}_{12}\right)\Delta t = m_1(\vec{v}_{f1} - \vec{v}_{01})$
  - Object 2: $\left(\vec{F}_{g2} + \vec{\bar{F}}_{21}\right)\Delta t = m_2(\vec{v}_{f2} - \vec{v}_{02})$
  - Add: $\left(\vec{F}_{g1} + \vec{F}_{g2} + \vec{\bar{F}}_{12} + \vec{\bar{F}}_{21}\right)\Delta t = (m_1\vec{v}_{f1} + m_2\vec{v}_{f2})$ -$(m_1\vec{v}_{01} + m_2\vec{v}_{02})$
  - $\vec{F}_{12} = -\vec{F}_{21} \rightarrow \vec{\bar{F}}_{12} + \vec{\bar{F}}_{21} = 0 \rightarrow \sum\vec{F}_{external}\Delta t = \vec{p}_f - \vec{p}_0$
  - If $\sum\vec{F}_{external} = 0 \rightarrow 0 = \vec{p}_f - \vec{p}_0 = 0 \rightarrow \vec{p}_f = \vec{p}_0$



The Principle of Conservation of Linear Momentum

The total linear momentum of an isolated system remains constant (is conserved). An isolated system is one for which the vector sum of the average external forces acting on the system is zero.

$$m_1\vec{v}_{01} + m_2\vec{v}_{02} = m_1\vec{v}_{f1} + m_2\vec{v}_{f2}$$

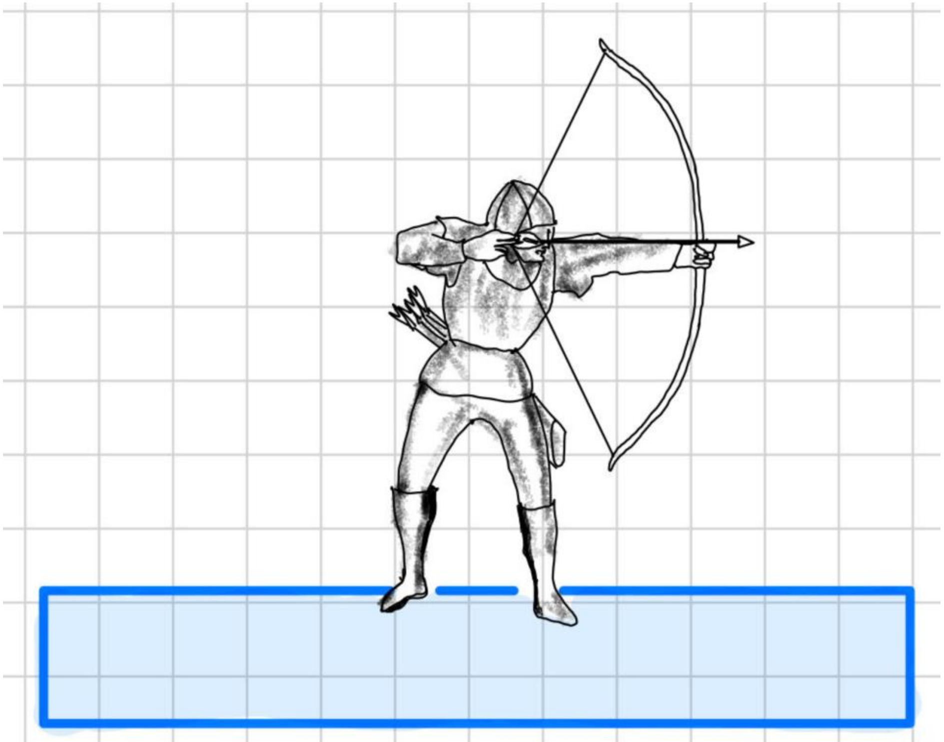
Example: Conservation of Linear Momentum 1/4

An archer stands at rest on frictionless ice; his total mass including his bow and quiver of arrows is 60 kg. His draw length is 0.800 m.

a. If the archer fires a 0.030 kg arrow horizontally at 50.0 m/s in the positive x direction, what is his subsequent velocity across the ice?

b. He then fires a second identical arrow at the same speed relative to the ground but at an angle of 30^0 above the horizontal. Find his new speed.

c. Estimate the average normal force acting on the archer as the second arrow is accelerated by the bowstring.



Example: Conservation of Linear Momentum 2/4

An archer stands at rest on frictionless ice; his total mass including his bow and quiver of arrows is 60 kg. His draw length is 0.800 m.

a. If the archer fires a 0.030 kg arrow horizontally at 50.0 m/s in the positive x direction, what is his subsequent velocity across the ice?

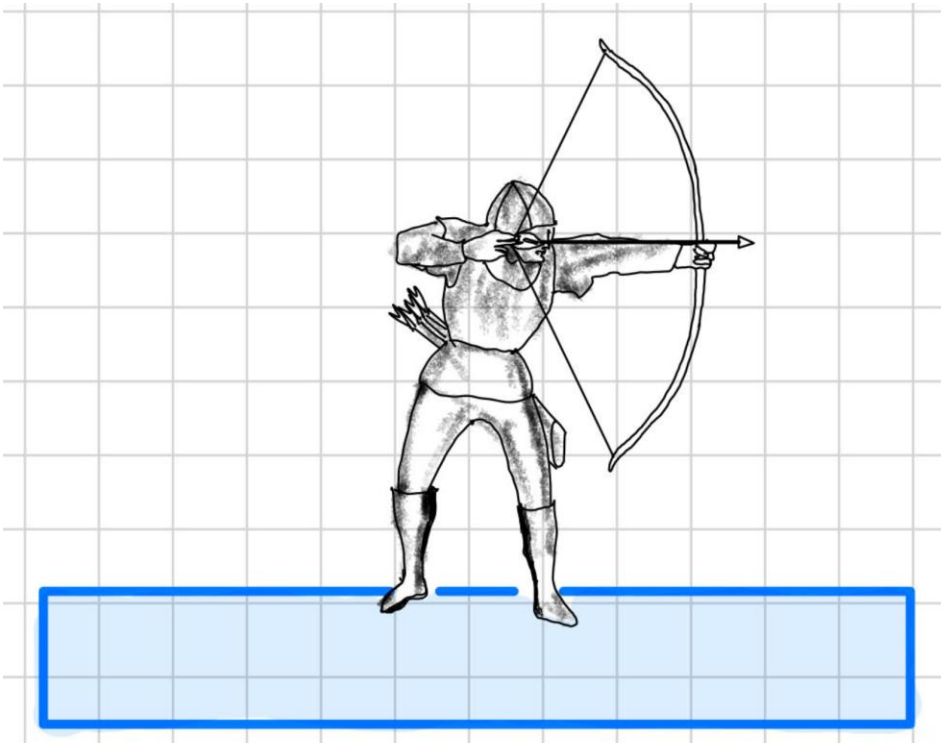
Solution: Start with the conservation of momentum,

$$p_{0x} = p_{fx}$$

Let m_1 and v_1 be the archer's mass and velocity after firing the arrow, respectively, and m_2 and v_{2f} the arrow's mass and velocity. Both velocities are in the x-direction. The initial velocities of both the arrow and archer are 0,

$$0 = (m_1 - m_2)v_{1f} + m_2 v_{2f}$$

$$v_{1f} = -\frac{m_2}{(m_1 - m_2)} v_{2f} = -\left(\frac{0.030\ kg}{59.97\ kg}\right)\left(50.0\frac{m}{s}\right) = -0.025\ m/s$$



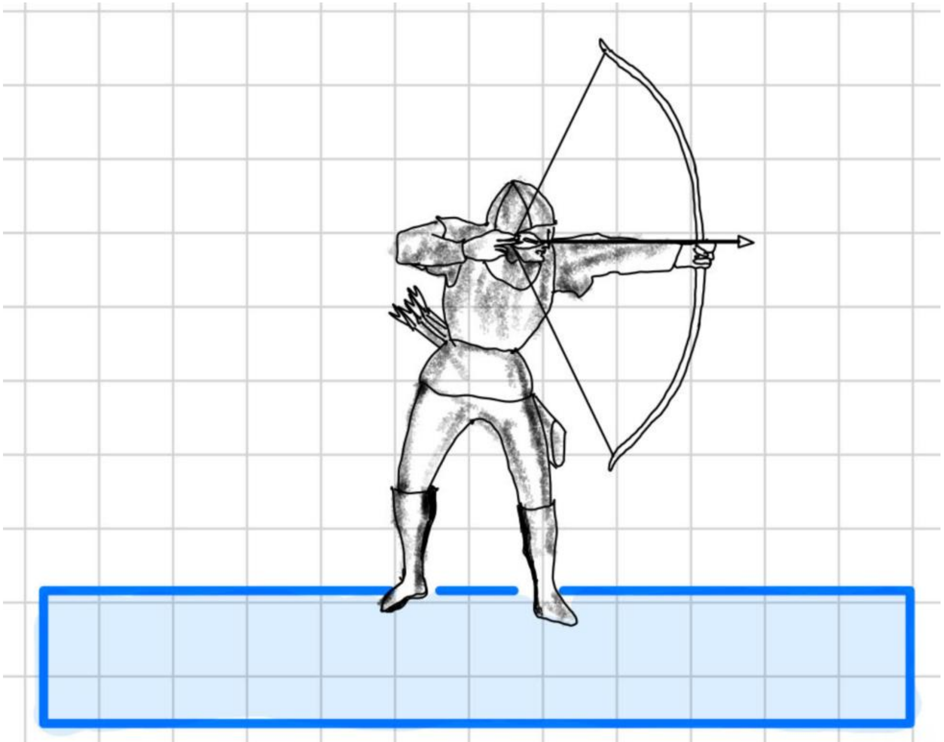
If the archer is using one of his arrows, his mass becomes:

$$m_1 - m_2 = (60.0 - 0.030)\ kg\ = 59.97\ kg$$

Example: Conservation of Linear Momentum 3/4

An archer stands at rest on frictionless ice; his total mass including his bow and quiver of arrows is 60 kg. His draw length is 0.800 m.

b. He then fires a second identical arrow at the same speed relative to the ground but at an angle of 30^0 above the horizontal. Find his new speed.



Solution: Again, the conservation of momentum, but this time the archer is still sliding from firing the first arrow,

$$m_1 v_{10} = (m_1 - 2m_2)v_{1f} + m_2 v_{2f}\cos\theta$$

$$v_{1f} = \frac{m_1}{m_1 - 2m_2}v_{10} - \frac{m_2}{m_1 - 2m_2}v_{2f}\cos\theta$$

$$= \left(\frac{59.97\ kg}{59.94\ kg}\right)(-0.025\ m/s) - \left(\frac{0.030\ kg}{59.94\ kg}\right)(50.0\ m/s)\cos(30^0)$$

$$v_{1f} = -0.0467\ m/s$$

Note we used the archer's final velocity from part a as the initial velocity.

If the archer is using a second one of his arrows, his mass becomes:
$m_1 - 2m_2 = (60.0 - 2 \cdot 0.030)\ kg\ = 59.94\ kg$

Example: Conservation of Linear Momentum 4/4

An archer stands at rest on frictionless ice; his total mass including his bow and quiver of arrows is 60 kg. His draw length is 0.800 m.

c. Estimate the average normal force acting on the archer as the second arrow is accelerated by the bowstring.

Solution: Use kinematics in 1D to estimate the acceleration of the arrow:

$$v^2 - v_0^2 = 2a\Delta x \rightarrow a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{(50\ m/s)^2 - 0}{2(0.800\ m)} = 1.56 \times 10^3\ m/s^2$$

Find Δt of the arrow: $v_{2f} = v_0 + a\Delta t$

$$\Delta t = \frac{v_{2f} - v_0}{a} = \frac{(50-0)m/s}{1.56 \times 10^3 m/s^2} = 0.032\ s$$

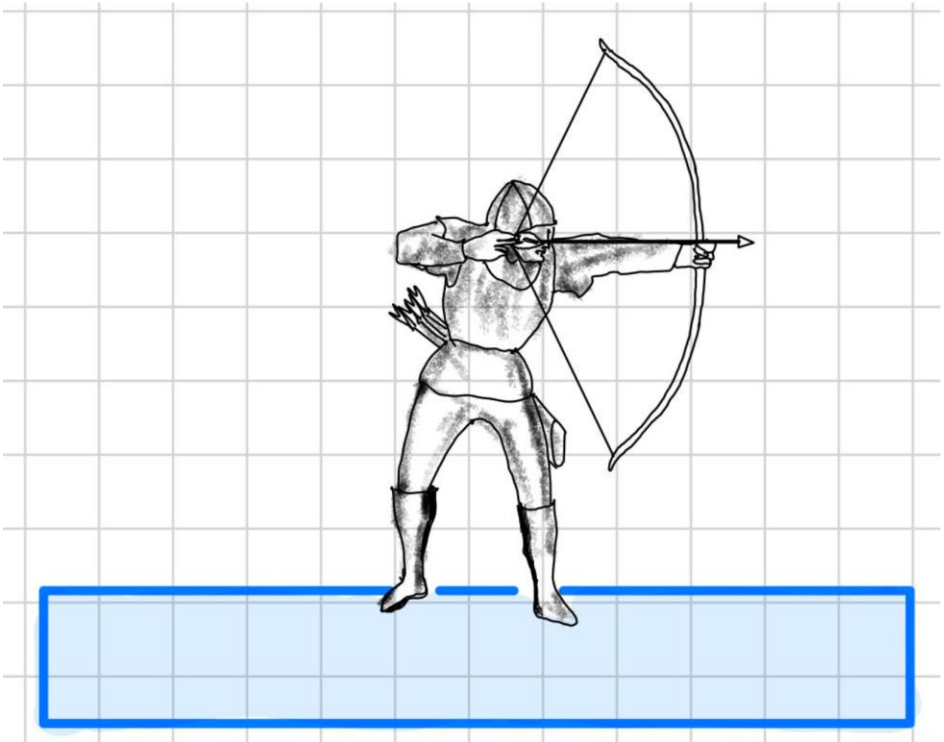
Now use the impulse momentum theorem in the y-direction,

$$\bar{F}_y \Delta t = \Delta p_y \rightarrow \ \bar{F}_y = \frac{\Delta p_y}{\Delta t} = \frac{m_2 v_{2f} \sin\theta}{\Delta t}$$

The average normal force is given by the archer's weight plus $\bar{F}_y$,

$$\sum F_y = F_N - m_1 g - \bar{F}_y = 0 \rightarrow F_N = m_1 g + \frac{m_2 v_{2f}\sin\theta}{\Delta t}$$

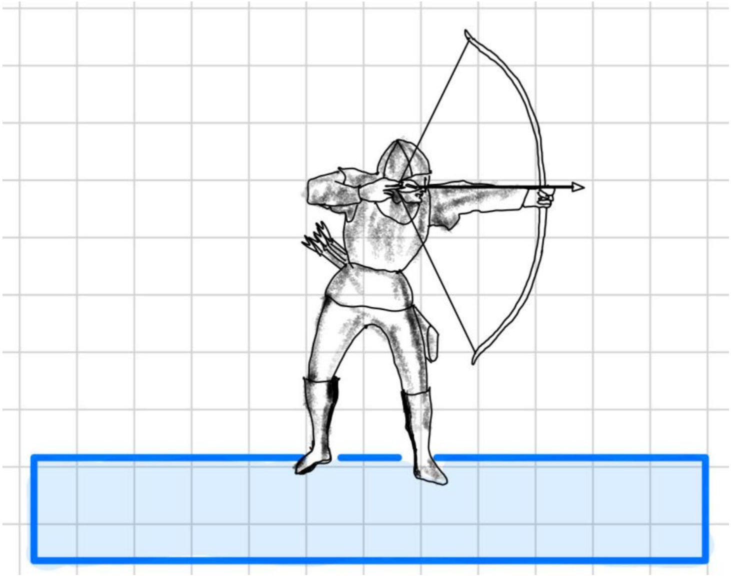
$$F_N = (59.94\ kg)(9.81\ ms^2) + \frac{(0.030\ kg)(50.0\ ms)\sin 30^0}{0.032\ s} = 6.11 \times 10^2 N$$



This problem is essentially a simplified version of rocket propulsion. The archer plus his remaining arrows is like a rocket plus its remaining fuel. Each arrow is a "fuel unit" ejected at a fixed speed relative to the rocket, and the archer's recoil after each shot comes from conservation of momentum. As the total mass decreases, each shot has a larger effect—exactly the same principle that governs the Tsiolkovsky rocket equation used in early spaceflight calculations.

Team Activity: Concept Check 7.2

In the previous problem, the archer fired an arrow twice. The first arrow he fired horizontally and the second at an angle to the horizontal. If the archer had fired both arrows horizontally, what would the new average normal force be?



Collisions in One Dimension

Collisions are classified according to whether the kinetic energy of the system changes during the collision:

1. **Elastic Collision**: One in which the total kinetic energy of the system after the collision is equal to the total kinetic energy before the collision: $KE_f = KE_0$
2. **Inelastic Collision**: One in which the total kinetic energy of the system is not the same before and after the collision: $KE_f \neq KE_0$

In either type of collision, momentum is conserved

$$\vec{p}_f = \vec{p}_0$$

Elastic Collisions

The figure shows an elastic head-on collision between two balls. No external forces act on the balls. What are the velocities of the balls after the collisions?

Solution: Elastic → $KE_f = KE_0$

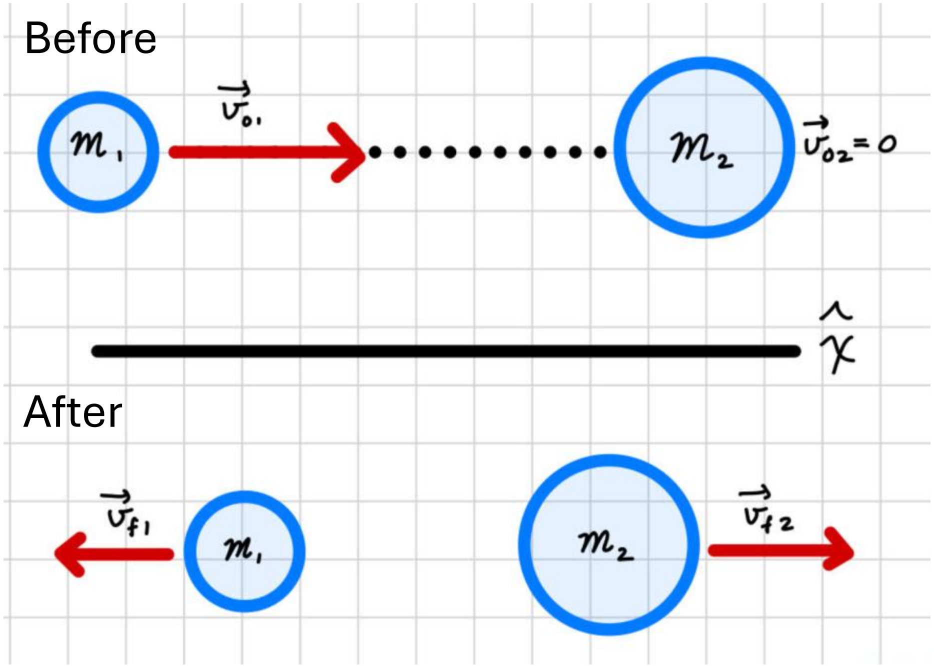
$$KE_f = \frac{1}{2}m_1 v_{f1}^2 + \frac{1}{2}m_2 v_{f2}^2 = \frac{1}{2}m_1 v_{01}^2 + \frac{1}{2}m_2 v_{02}^2 = KE_0$$

$$\boxed{v_{f1}^2 = v_{01}^2 - \frac{m_2}{m_1}\left(v_{f2}^2 - v_{02}^2\right)}$$

Now use the conservation of momentum to get a value for v_{f2}:

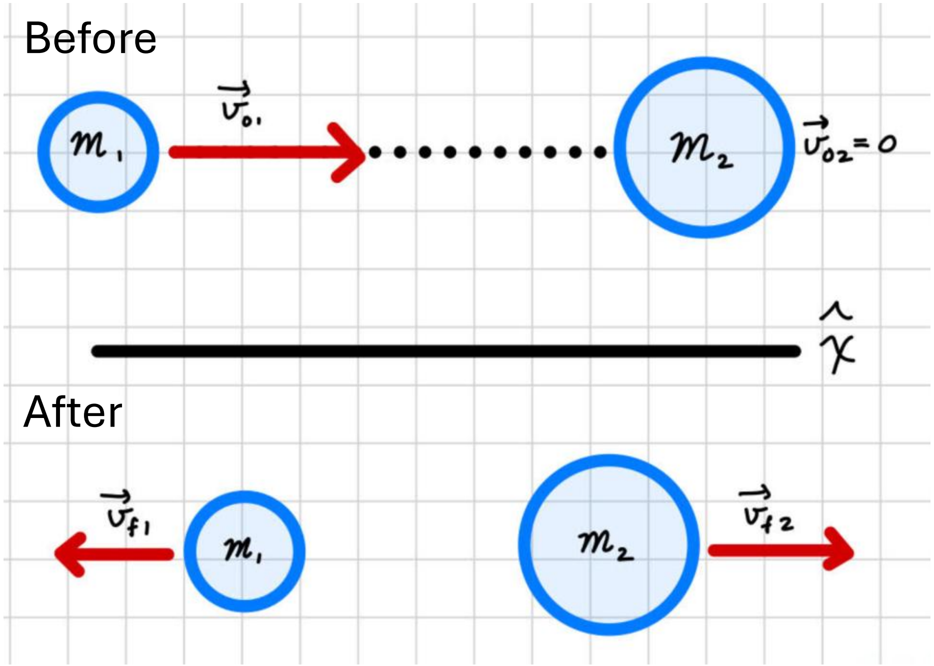
$$m_1 v_{f1} + m_2 v_{f2} = m_1 v_{01} + m_2 v_{02}$$

$$\boxed{v_{f2} = \frac{m_1}{m_2}\left(v_{01} - v_{f1}\right) + v_{02}}$$



Example: Elastic Collisions

The figure shows an elastic head-on collision between two balls. No external forces act on the balls. One ball has a mass of $m_1 = 0.250\ kg$ and an initial velocity of $v_{01} = 5.00\ m/s$ in the $\hat{x}$ direction. The other, $m_2 = 0.80\ kg$ and is initially at rest.



What are the velocities of the balls after the collisions?

Solution: Use the two equations from the previous slide

$$v_{f1}^2 = v_{01}^2 - \frac{m_2}{m_1}\left(v_{f2}^2 - \cancel{v_{02}^2}\right) = v_{01}^2 - \frac{m_2}{m_1} v_{f2}^2 \qquad [1]$$

$$v_{f2} = \frac{m_1}{m_2}(v_{01} - v_{f1}) + \cancel{v_{02}} = \frac{m_1}{m_2}(v_{01} - v_{f1}) \rightarrow v_{f2}^2 = \left(\frac{m_1}{m_2}\right)^2 (v_{01} - v_{f1})^2 \qquad [2]$$

Substitute [2] into [1],

$$v_{f1}^2 = v_{01}^2 - \frac{m_2}{m_1}\left(\frac{m_1}{m_2}\right)^2 (v_{01} - v_{f1})^2 \rightarrow v_{01}^2 - v_{f1}^2 = \frac{m_1}{m_2}(v_{01} - v_{f1})^2$$

$$(v_{01} - v_{f1})(v_{01} + v_{f1}) = \frac{m_1}{m_2}(v_{01} - v_{f1})(v_{01} - v_{f1})$$

$$v_{01} + v_{f1} = \frac{m_1}{m_2}(v_{01} - v_{f1}) \rightarrow v_{f1}\left(\frac{m_1}{m_2} + 1\right) = v_{01}\left(\frac{m_1}{m_2} - 1\right)$$

$$v_{f1} = v_{01}\frac{\left(\frac{m_1}{m_2} - 1\right)}{\left(\frac{m_1}{m_2} + 1\right)} = v_{01}\left(\frac{m_1 - m_2}{m_1 + m_2}\right) \rightarrow \boxed{v_{f1} = v_{01}\left(\frac{m_1 - m_2}{m_1 + m_2}\right)}$$

$$v_{f2} = \frac{m_1}{m_2}(v_{01} - v_{f1}) = v_{01}\left(\frac{2m_1m_2}{m_1 + m_2}\right) \rightarrow \boxed{v_{f2} = v_{01}\left(\frac{2m_1m_2}{m_1 + m_2}\right)}$$

$$v_{f1} = (5.00\ m/s)\left(\frac{0.250 - 0.80}{0.250 + 0.80}\right) = -2.62\ m/s$$

$$v_{f2} = (5.00\ m/s)\left(\frac{2 \cdot 0.250 \cdot 0.80}{0.250 + 0.80}\right) = 2.38\ m/s$$

What if they stick together?

Two balls, m_1 and m_2, are on a collision course with velocities $\vec{v}_1$ and $\vec{v}_2$. After colliding, through some unknown process become a single larger ball with mass $M = m_1 + m_2$ and velocity $\vec{V}$.

The conservation of momentum tells us that, $\vec{p}_f = \vec{p}_0$, so in the $\hat{x}$ direction we know that,

$$p_f = (m_1 + m_2)V = m_1 v_1 + m_2 v_2 = p_0 \rightarrow V = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Now let's look at the kinetic energy,

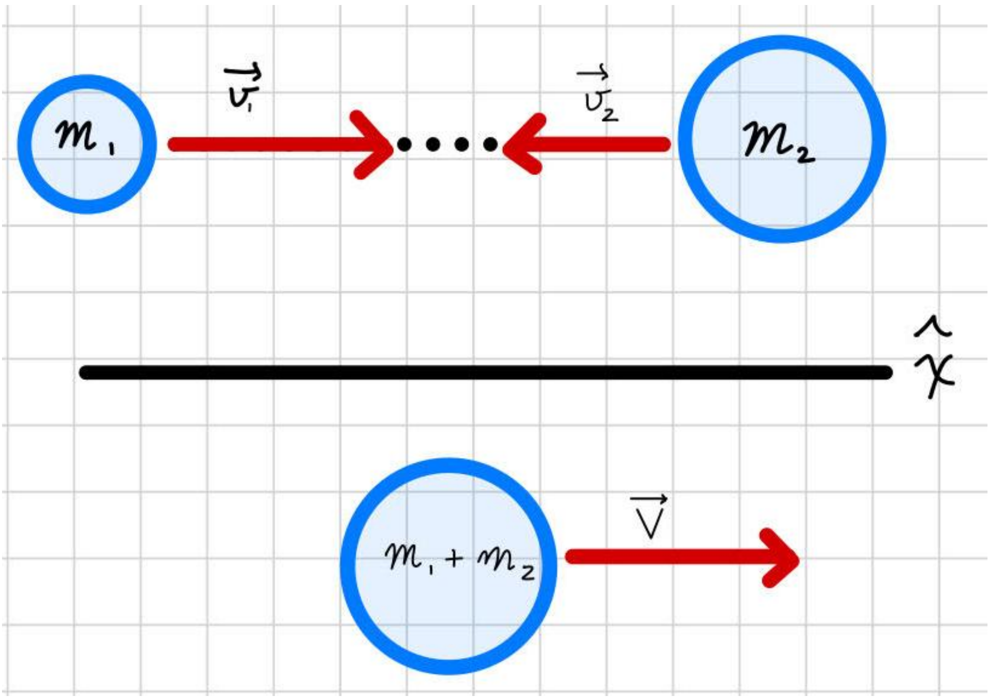
$$KE_f = \frac{1}{2}(m_1 + m_2)V^2 \rightarrow KE_f = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\right)^2$$

$$KE_F = \frac{(m_1 v_1 + m_2 v_2)^2}{2(m_1 + m_2)}$$

$$KE_0 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

$$KE_0 - KE_f = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(v_1 - v_2)^2$$

This final expression means that $KE_0 \neq KE_f$ unless $v_1 = v_2$!



If two objects collided then $v_1 \neq v_2$ otherwise there could be no collision, momentum change, or change in kinetic energy! So, if two objects collide and stick together the collision is **completely inelastic**! Whatever process allowed the masses to stick together used kinetic energy resulting in the inelastic condition: $KE_0 \neq KE_f$

Example: Inelastic Collisions

A ballistic pendulum can be used to measure the speed of a projectile, such as a bullet. The ballistic pendulum consists of a stationary block of wood with mass m_2 suspended by a wire of negligible mass. A bullet with mass m_1 is fired into the block, and the block (now with the bullet in it) swings to a maximum height of h_f above the initial position. Find the speed at which the bullet is fired, assuming that air resistance is negligible.

Solution: Since the bullet and block stick after the collision, it is completely inelastic. Momentum is still conserved,

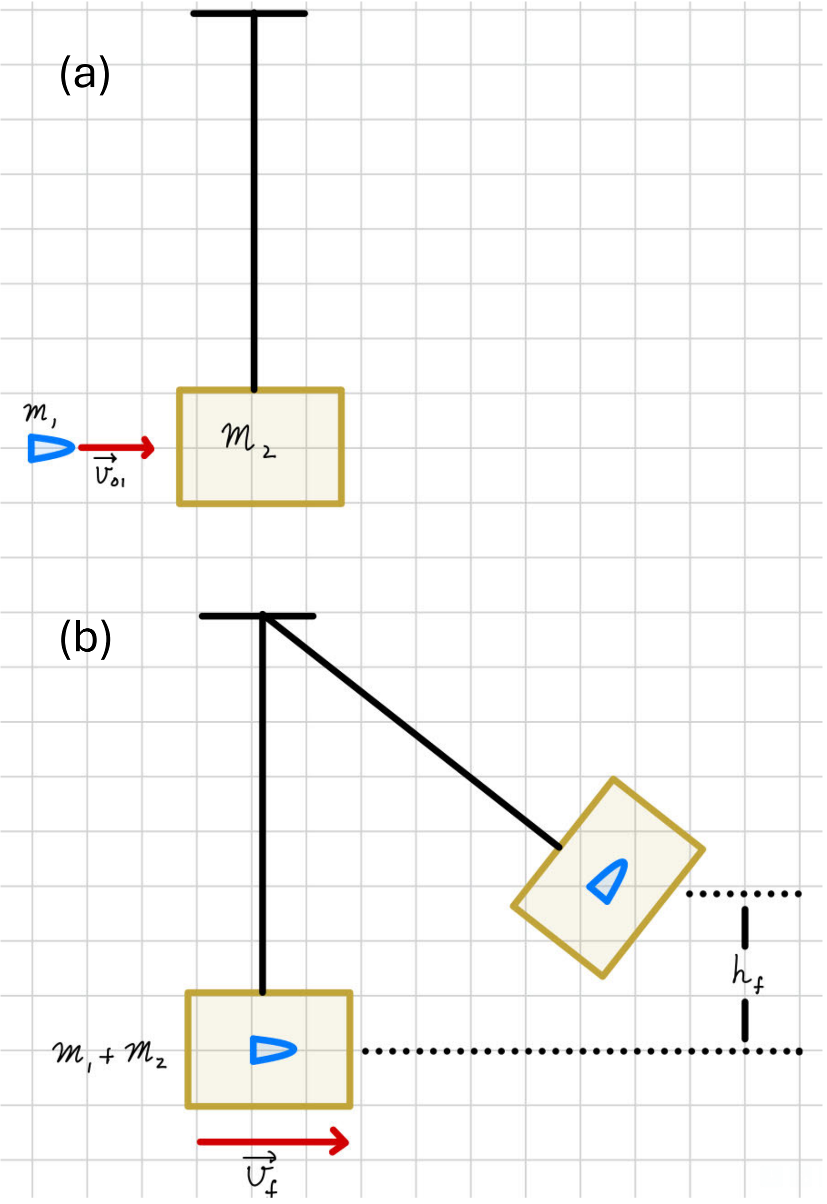
$$p_f = (m_1 + m_2)v_f = m_1 v_{01} = p_0 \rightarrow v_{01} = \left(\frac{m_1 + m_2}{m_1}\right) v_f$$

While we can't use conservation of energy on the bullet, we can use it on the block once the bullet is lodged in it since nonconservative forces do no work as they are perpendicular to the direction of motion.

$$PE = (m_1 + m_2)h_f = \frac{1}{2}(m_1 + m_2)v_f^2 = KE \rightarrow v_g = \sqrt{2gh_f}$$

$$v_{01} = \left(\frac{m_1 + m_2}{m_1}\right)\sqrt{2gh_f}$$

Note: It is tempting to say that the total PE at the top of the swing is equal to the KE of the bullet just before the collision, but it is nonelastic so that would not work as some KE is spent in the collision when the bullet sticks to the block.

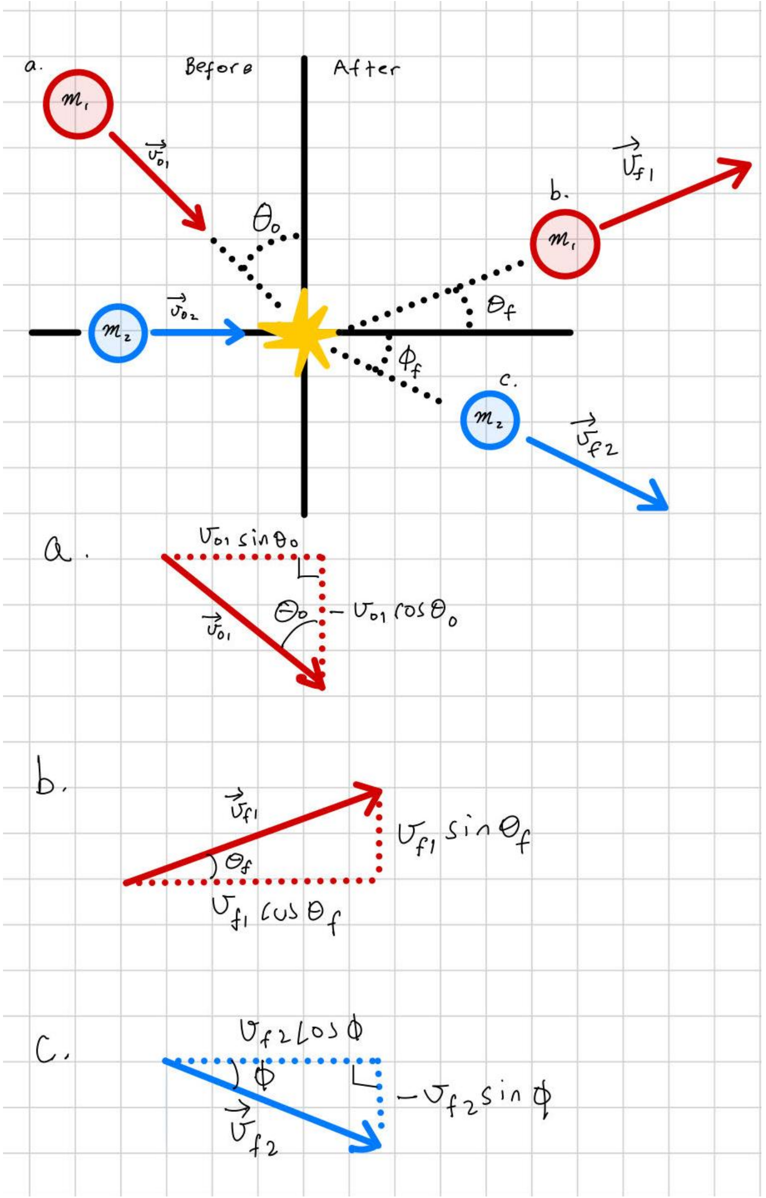


Problem Solving Insight: Collisions

Elastic Collisions	Inelastic Collisions
$\vec{p}_f = \vec{p}_0$ $KE_f = KE_0$ $E_f = E_0$	$\vec{p}_f = \vec{p}_0$ $KE_f \neq KE_0$ $E_f \neq E_0$ Note: *Often the problem can be split into two parts. Before and after the collision. Use the conservation of momentum before the collision, and after the collision, the resulting motion can often be described with the conservation of mechanical energy.*

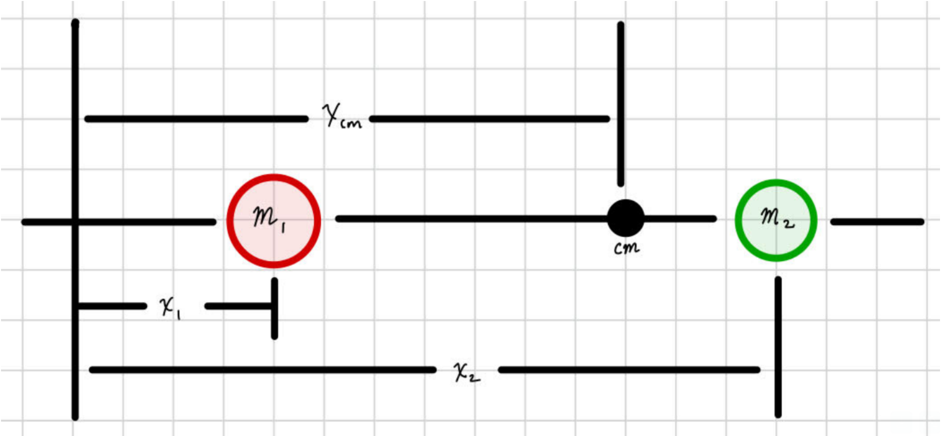
Collisions in Two Dimensions

- Linear momentum is a vector, and just like all vectors it can be broken down into its individual components. If there are velocity components in x and y then you can write the conservation of momentum for both,
  - x-component: $m_1 v_{f1x} + m_2 v_{f2x} = m_1 v_{01x} + m_2 v_{02x}$
  - y-component: $m_1 v_{f1y} + m_2 v_{f2y} = m_1 v_{01y} + m_2 v_{02y}$
- For the diagram on the right,
  - x-component: $m_1 v_{f1} \cos\theta_f + m_2 v_{f2} \cos\phi_f = m_1 v_{01} \sin\theta_0 + m_2 v_{02}$
  - y-component: $m_1 v_{f1} \sin\theta_f - m_2 v_{f2} \sin\phi_f = -m v_{01} \cos\theta_0$



Center of Mass

If you've ever seen someone do a cartwheel or toss a baton, you might think their motion is chaotic. But in physics, we can find one special point that behaves in a very predictable, simple way, ignoring all the spinning and flailing, and that's the center of mass. It's like the quiet, orderly leader of a marching band, walking in a straight line while the rest of the group dances around them.



- The center of mass is a point that represents the average location for the total mass of a system.
- **Why Do We Care About the Center of Mass?**
  - **Simplifies Complex Motion**: When dealing with complicated systems such as spinning, breaking, tumbling objects, it lets us treat them *as if* all the mass were concentrated at a single point. This means we can apply Newton's laws to the motion of the system using just the center of mass.
  - **Connects to Real-World Intuition**:
    - When you throw a wrench, it spins, but its center of mass moves like a regular projectile.
    - When a car crashes, investigators look at the center of mass to reconstruct the motion.
    - When rockets launch, they balance fuel tanks to keep the center of mass in line with the thrust.
  - **Necessary for Conservation Laws**: If no external force acts on a system, the center of mass doesn't accelerate.
  - Consider the diagram,

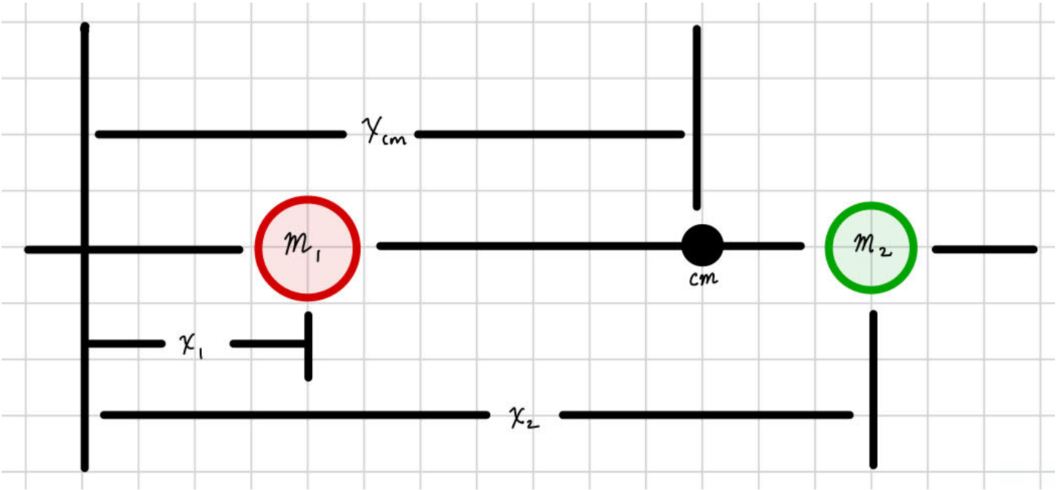
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \rightarrow \boxed{x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}}$$

Generalized Form

If $m_1 = m_2 = m$,

$$x_{cm} = \frac{m(x_1 + x_2)}{2m} = \frac{1}{2}(x_1 + x_2)$$

Team Activity: Concept Check 7.3



In the diagram, which is not drawn to scale, if $m_1 > m_2$, do you expect the center-of-mass to be to the left or right of the midpoint and why?

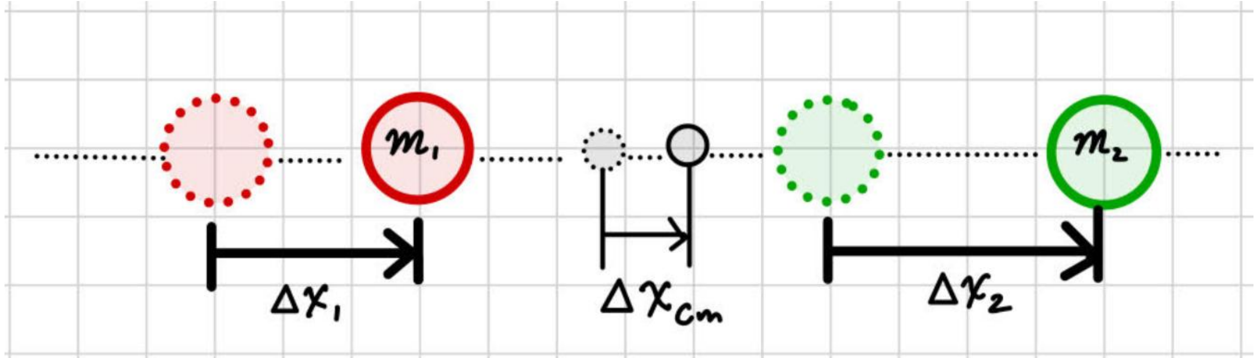
Change in the Center-of-Mass

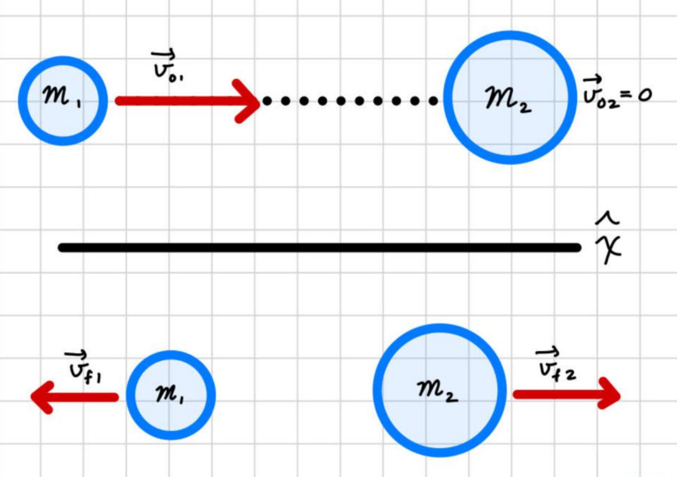
Displacement:

- $\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$
- General: $\Delta x_{cm} = \frac{\sum_{i=1}^{n} m_i \Delta x_i}{\sum_{i=1}^{n} m_i}$

Velocity:

- $v_{cm} = \lim_{\Delta t \to \infty} \frac{\Delta x_{cm}}{\Delta t} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$
- General: $v_{cm} = \frac{\sum_{i=1}^{n} m_i v_i}{\sum_{i=1}^{n} m_i}$





Elastic. One ball has a mass of $m_1 = 0.250\ kg$ and an initial velocity of $v_{01} = 5.00\ m/s$ in the $\hat{x}$ direction. The other, $m_2 = 0.80\ kg$ and is initially at rest. Recall we calculated $v_{f1} = -2.62\ m/s$ and $v_{f2} = 2.38\ m/s$

Revisit previous example.

Before: $v_{cm} = \frac{(0.250\ kg)(5.00\ m/s) + (0.800\ kg)(0\ m/s)}{(0.250 + 0.800) kg} = 1.19\ m/s$

After: $v_{cm} = \frac{(0.250\ kg)(-2.62\ m/s) + (0.800\ kg)(2.38\ ms)}{(0.250 + 0.800) kg} = 1.19\ m/s$

The velocity of the center of mass is the same before and after objects interact during an elastic collision!

Example: Center-of-Mass 1/4

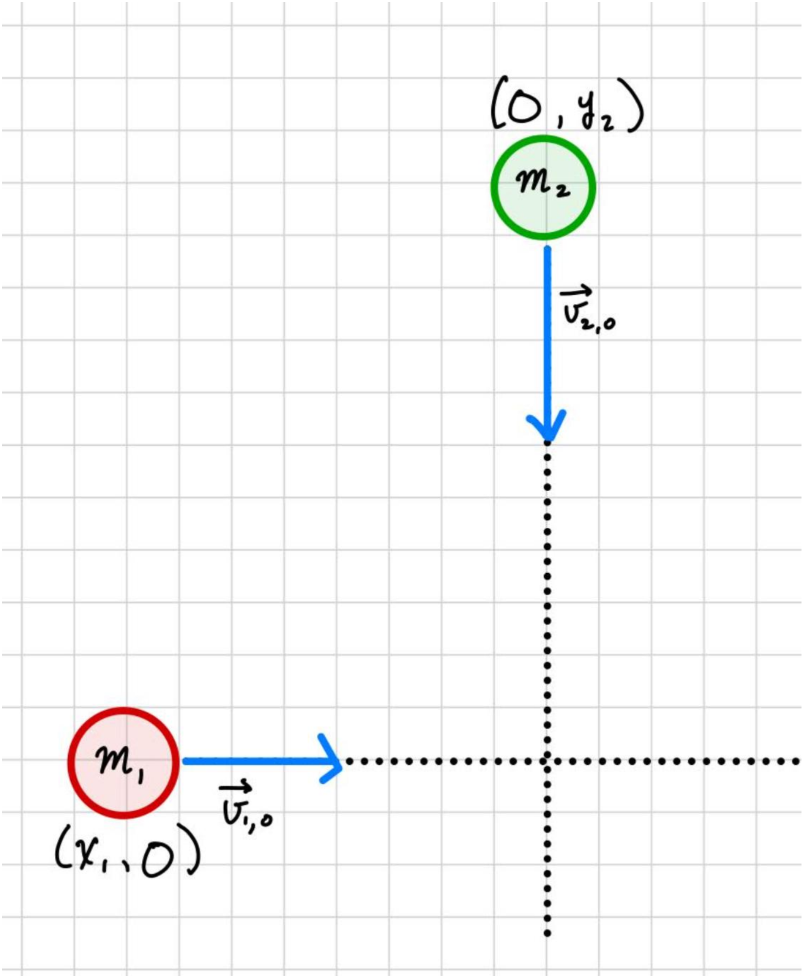
Two objects undergo an elastic collision. Object 1is a 5 kg ball moving along the x-axis, and object 2 is a 3 kg ball moving along the y-axis. The initial positions and velocities of the objects:

$$(x_1, y_1) = (2{,}0)m, \vec{v}_{10} = (3{,}0)m/s$$
$$(x_2, y_2) = (0{,}4)m, \overrightarrow{v}_{2,0} = (0, -2)m/s$$

a. Calculate the initial position of the center of mass (x_{cm}, y_{cm}) of the system.

b. Determine the velocity of the center-of-mass $\vec{v}_{cm,0}$ of the system before the collision.

c. What is the final velocities of the two objects after the collision?



Example: Center-of-Mass 2/4

Two objects undergo an elastic collision. Object 1is a 5 kg ball moving along the x-axis, and object 2 is a 3 kg ball moving along the y-axis. The initial positions and velocities of the objects:

$$(x_1, y_1) = (2{,}0)m, \vec{v}_{10} = (3{,}0)m/s$$

$$(x_2, y_2) = (0{,}4)m, \vec{v}_{2,0} = (0, -2)m/s$$

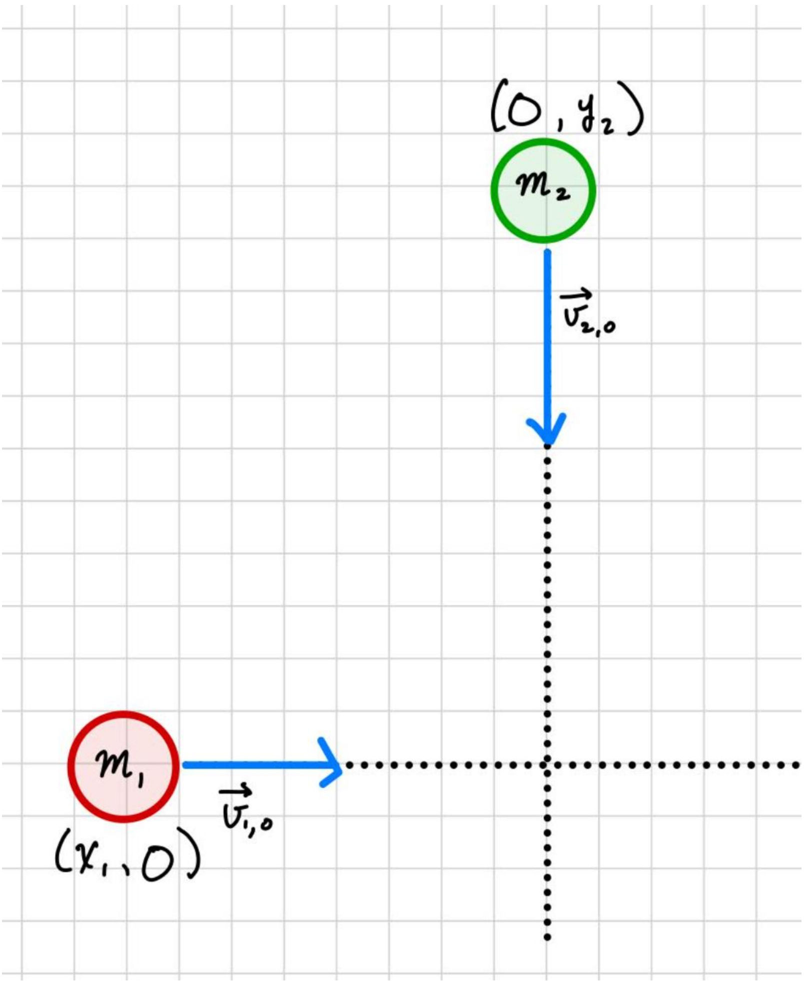
a. Calculate the initial position of the center of mass (x_{cm}, y_{cm}) of the system.

Solution: Calculate x_{cm} and y_{cm}.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(5\ kg)(2\ m)}{(5+3)kg} = 1.25m$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{(3\ kg)(4\ m)}{(5+3)kg} = 1.5\ m$$

$$(x_{cm}, y_{cm}) = (1.25, 1.5)\ m$$



Example: Center-of-Mass 3/4

Two objects undergo an elastic collision. Object 1is a 5 kg ball moving along the x-axis, and object 2 is a 3 kg ball moving along the y-axis. The initial positions and velocities of the objects:

$$(x_1, y_1) = (2{,}0)m, \vec{v}_{10} = (3{,}0)m/s$$

$$(x_2, y_2) = (0{,}4)m, \overrightarrow{v}_{2,0} = (0, -2)m/s$$

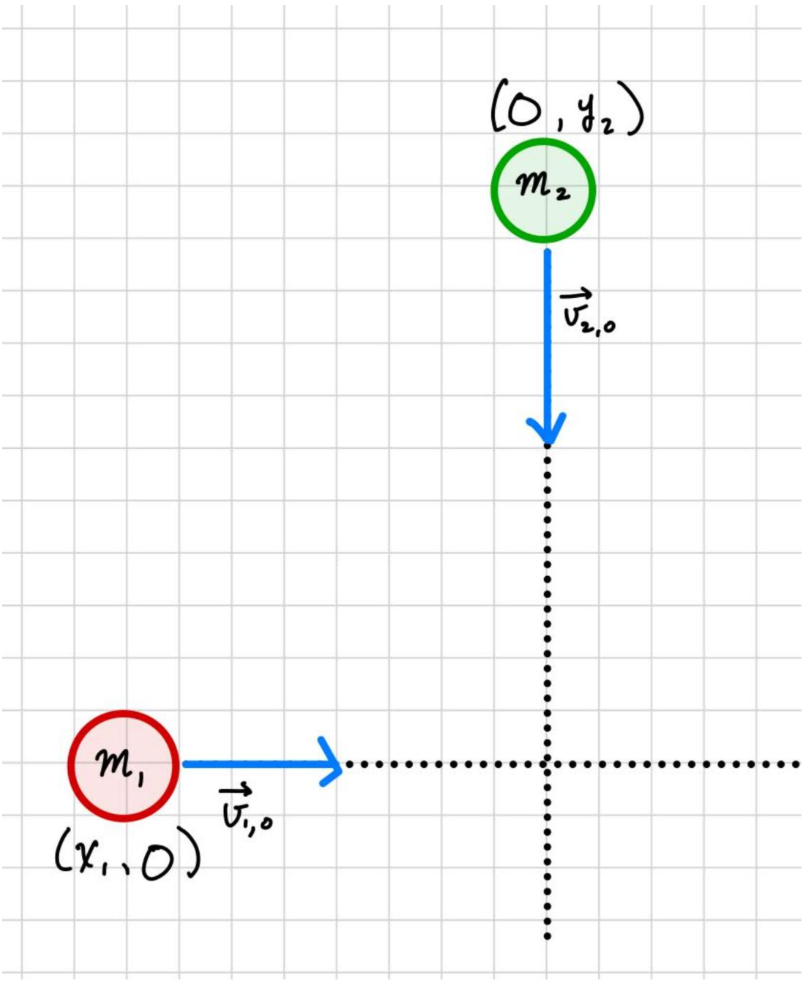
b. Determine the velocity of the center-of-mass $\vec{v}_{cm,0}$ of the system before the collision.

Solution: Again, compute the x and y components.

$$v_{cm_{x0}} = \left(\frac{m_1 v_{1,0} + m_2 v_{2,0}}{m_1 + m_2}\right)_x = \frac{(5\,kg)(3\,m/s)}{(5+3)kg} \approx 1.88\,m/s$$

$$v_{cm_{y0}} = \left(\frac{m_1 v_{1,0} + m_2 v_{2,0}}{m_1 + m_2}\right)_y = \frac{(3\,kg)(-2\,m/s)}{(5+3)kg} = -0.75\,m/s$$

$$\vec{v}_{cm} = (1.88, -0.75)m/s = (1.88\,\hat{x} - 0.75\,\hat{y})\,m/s$$



Example: Center-of-Mass 4/4

Two objects undergo an elastic collision. Object 1is a 5 kg ball moving along the x-axis, and object 2 is a 3 kg ball moving along the y-axis. The initial positions and velocities of the objects:

$$(x_1, y_1) = (2{,}0)m, \vec{v}_{10} = (3{,}0)m/s$$
$$(x_2, y_2) = (0{,}4)m, \vec{v}_{2,0} = (0, -2)m/s$$

c. What is $\vec{v}\_cm_f$ after the collision?

SURPRISE TEAM ACTIVITY: Concept Check 7.4