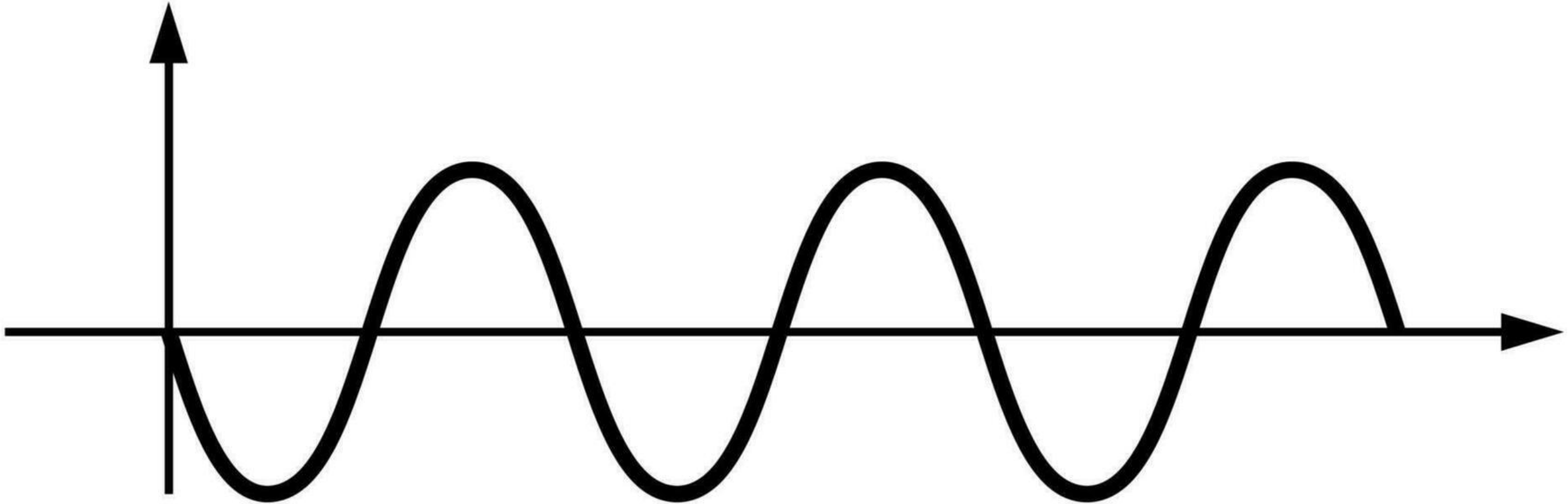
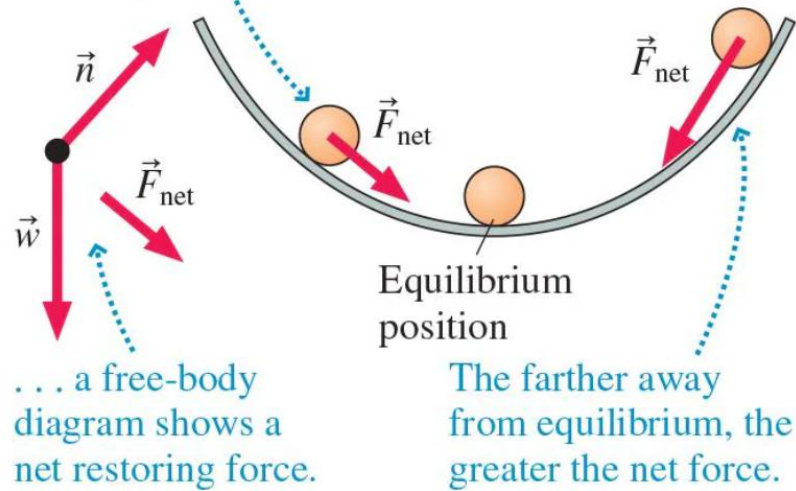


Restoring Forces, Simple Harmonic Motion, and Elasticity

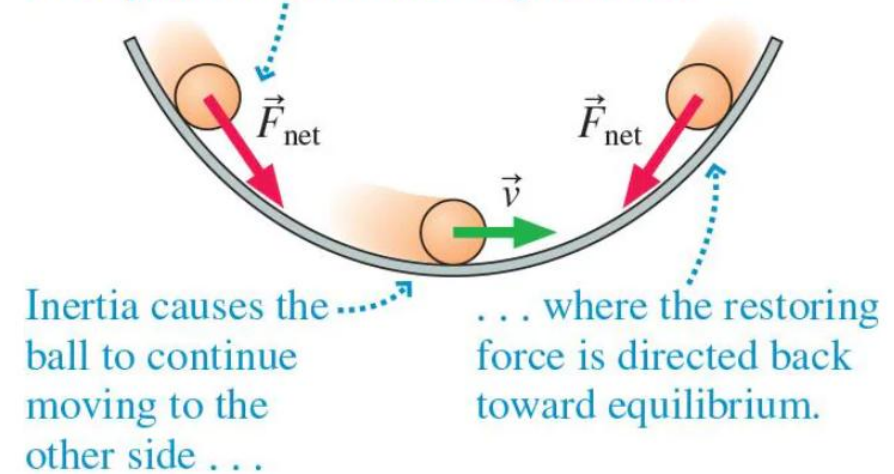


Equilibrium and Oscillation

When the ball is displaced from equilibrium . . .



When the ball is released, a restoring force pulls it back toward equilibrium.



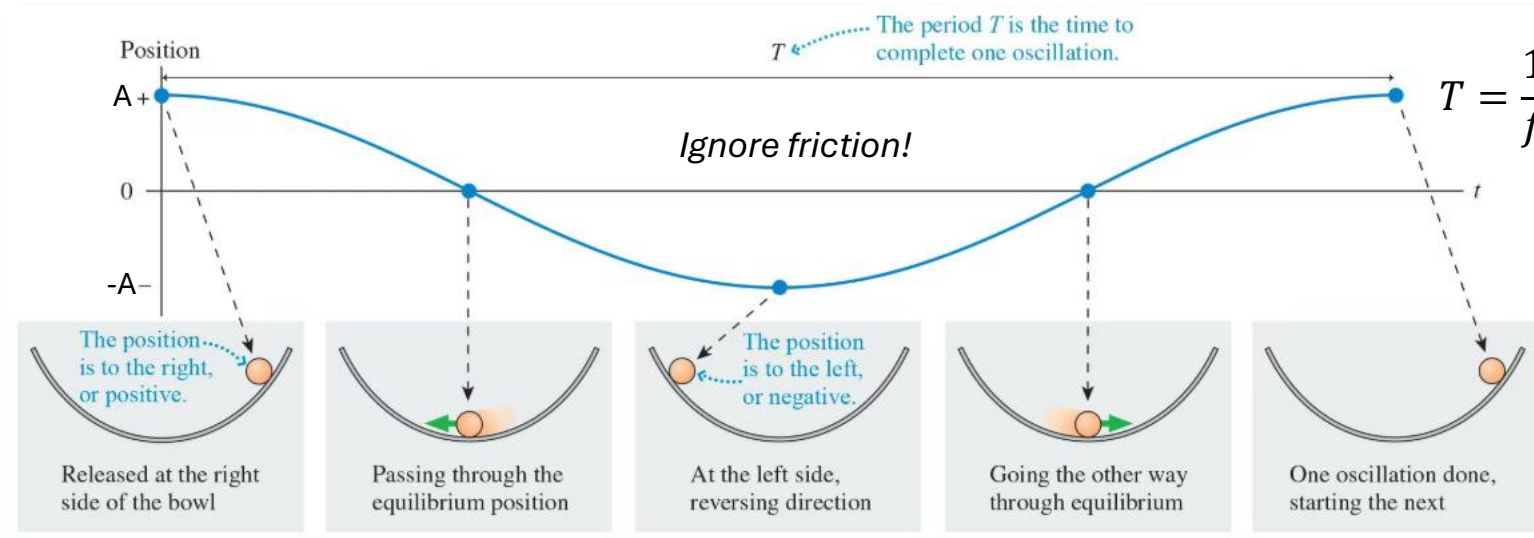
Consider a marble that is free to roll inside a spherical bowl. The marble has an **equilibrium position** at the bottom of the bowl where it will rest with no net force. If you push the marble away from equilibrium, the marble's weight leads to a net force directed back toward the equilibrium position. We call this a **restoring force**. Notice that the magnitude of this force increases as the marble is moved further away from the equilibrium position.

If you pull the marble to the side and release it, it doesn't just roll back to the bottom of the bowl and stay put. It keeps on moving, rolling up and down each side of the bowl, repeatedly moving through its equilibrium position. We call such a repetitive motion an **oscillation**. This oscillation is a result of an interplay between the restoring force and the marble's inertia.

Frequency and Period

Motion that is sinusoidal is called:

Simple Harmonic Motion

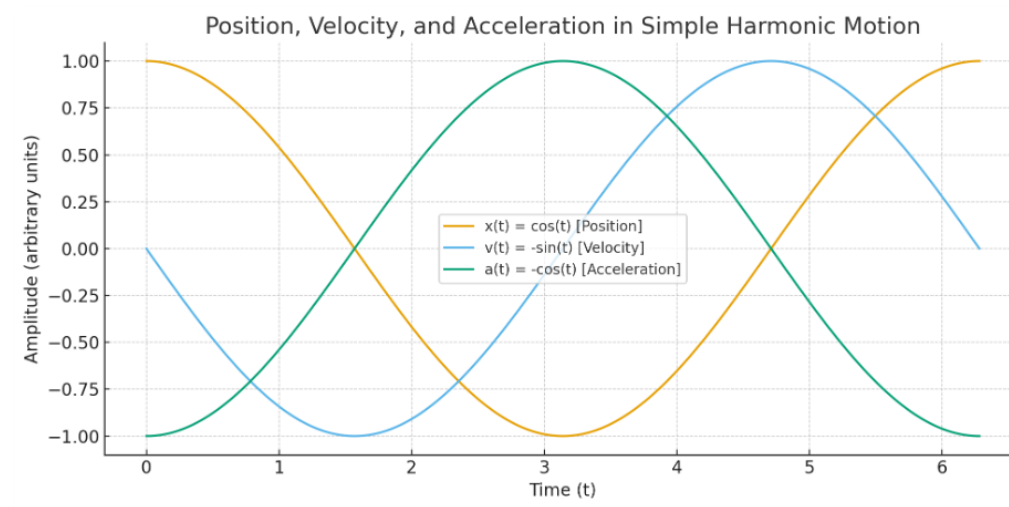


$$x(t) = A \cos(\omega t)$$

$$v(t) = -A\omega \sin(\omega t)$$

$$a(t) = -A\omega^2 \cos(\omega t)$$

Let $A = \omega = 1 \longrightarrow$



Example: Understanding the motion of a glider on a spring

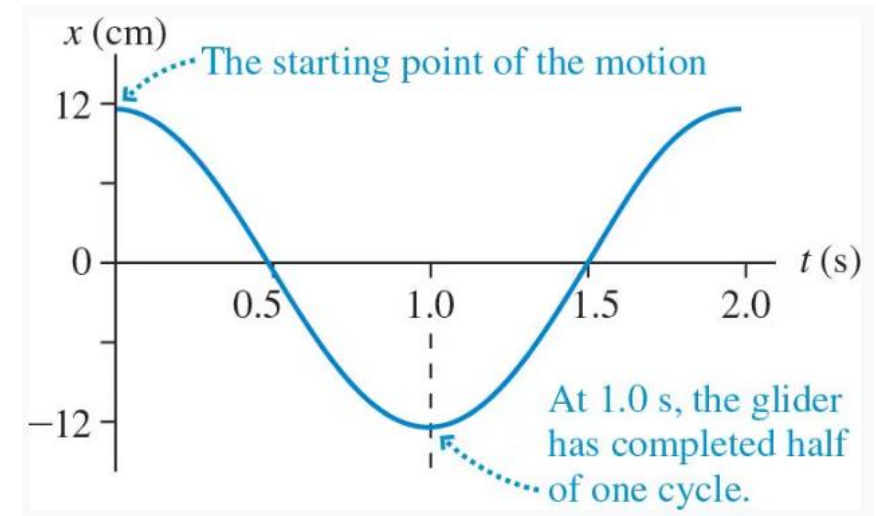
An air-track glider oscillates horizontally on a spring at a frequency of 0.50 Hz. Suppose the glider is pulled to the right of its equilibrium position by 12 cm and then released. Where will the glider be 1.0 s after its release? What is its velocity at this point?

Solution: The frequency is $f = 0.50 \text{ Hz}$, therefore the period is,

$$T = \frac{1}{f} = \frac{1}{0.50} \text{ s} = 2.0 \text{ s}$$

The maximum amplitude, or displacement, is 12.0 cm, and it initially occurs at $t = 0 \text{ s}$.

1.0 s is exactly half of the period. As the graph of the motion shows, half of the cycle brings the glider to its left turning point, 12 cm to the left of equilibrium. The velocity at this point is zero.



Linear Restoring Forces 1/2

Consider a glider on a track without friction. The glider is connected to the end of the track by a spring.

We displace the glider by Δx . The spring stretches, gets stiffer, and wants to compress, which would pull the glider to the left. Just like our marble in a bowl, there is a force that tries to pull it back toward its equilibrium position.

$$\vec{F}_{sp} = k\Delta\vec{x}$$

Here k is the spring constant and depends on the spring. Notice due to the way we have defined direction that if we compress the spring, $\Delta x < 0$, and if we stretch the spring, $\Delta x > 0$.

If we write the above equation in terms of the component of the spring force, we get the relationship between the **restoring force** and the displacement which is known as **Hooke's Law**:

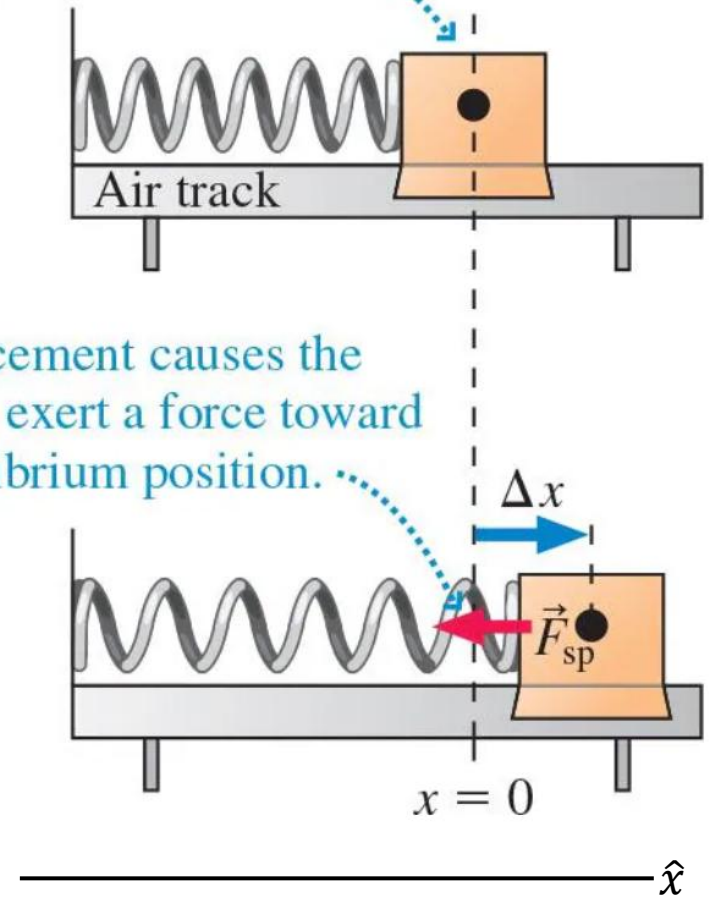
$$F_x = -k\Delta x$$

The negative sign tells us that this is a restoring force because the force is in the direction opposite the displacement. If we pull the glider to the right (x is positive), the force is to the left (negative)—back toward equilibrium.

It is linear because Δx is to the 1 power.

At equilibrium there is no net force.

A displacement causes the spring to exert a force toward the equilibrium position.



Linear Restoring Forces 2/2

Consider a glider on a track without friction. The glider is connected to the end of the track by a spring.

We displace the glider by Δx . The spring stretches, gets stiffer, and wants to compress, which would pull the glider to the left. Just like our marble in a bowl, there is a force that tries to pull it back toward its equilibrium position.

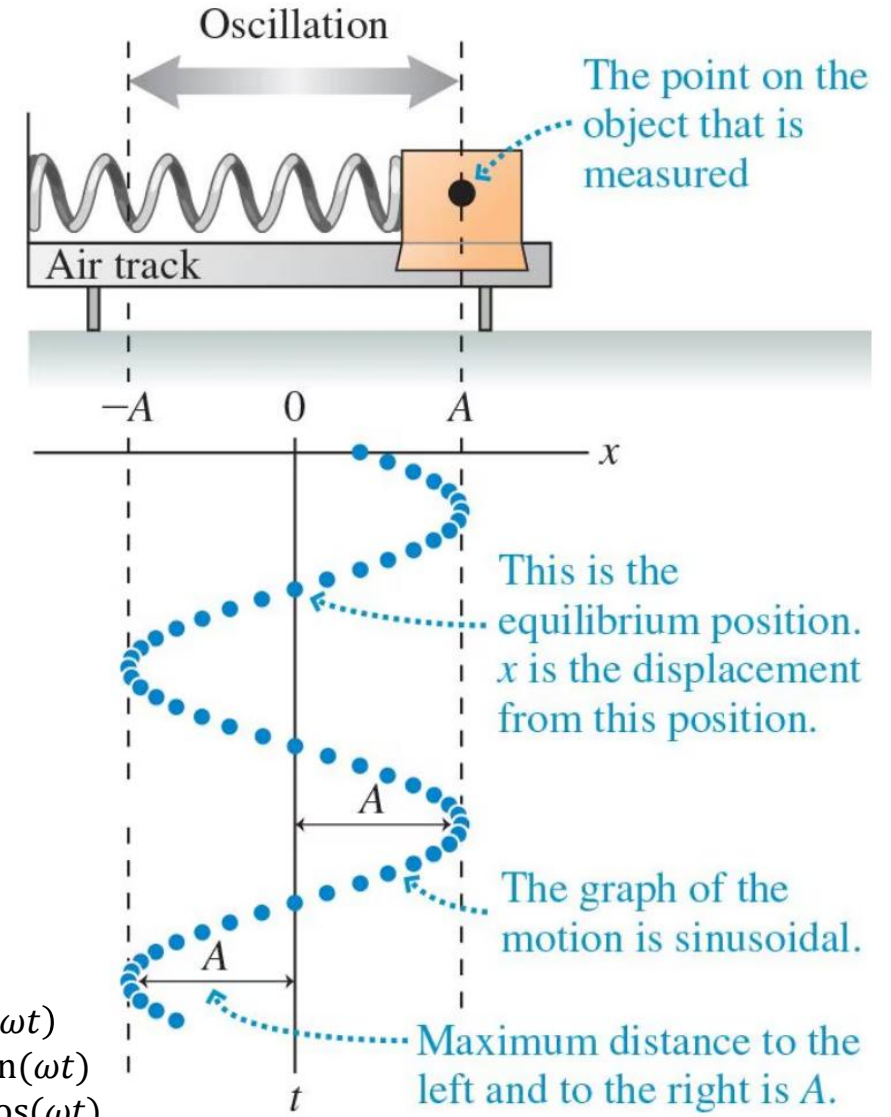
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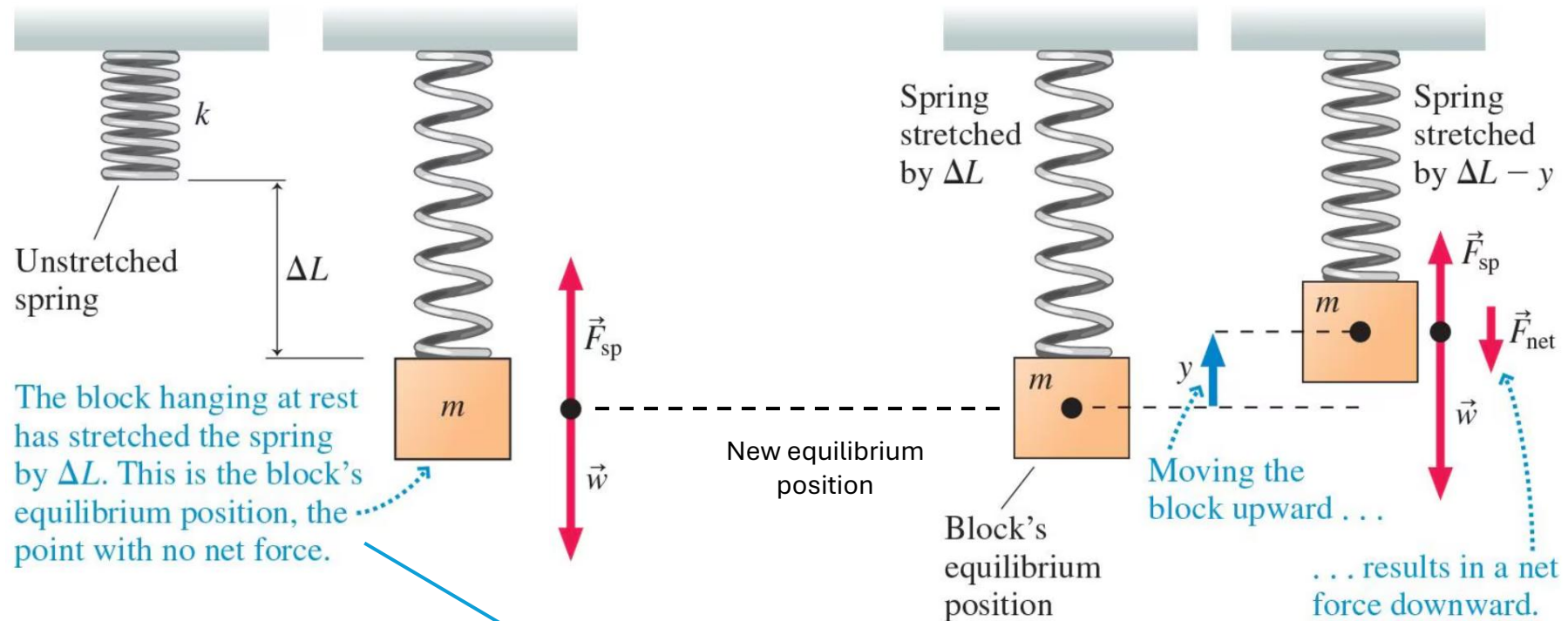
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$$F_x = -k\Delta x$$

The negative sign tells us that this is a restoring force because the force is in the direction opposite the displacement. If we pull the glider to the right (x is positive), the force is to the left (negative)—back toward equilibrium.



Vertical Motion of a Mass on a Spring



$$\sum F_y = (F_{sp})_y + w_y = k\Delta L - mg = 0 \rightarrow \Delta L = \frac{mg}{k}$$

The role of gravity is to determine where the equilibrium position is, but it doesn't affect the restoring force for displacement from the equilibrium position. Because it has a linear restoring force, a mass on a vertical spring oscillates with **simple harmonic motion!**

When the block is at position y , the spring is compressed by an amount $\Delta L - y$:

$$\begin{aligned} \sum F_y &= (F_{sp})_y + w_y = k(\Delta L - y) - mg = (k\Delta L - mg) - ky \\ &= \left(k \left(\frac{mg}{k} \right) - mg \right) - ky = -ky \end{aligned}$$

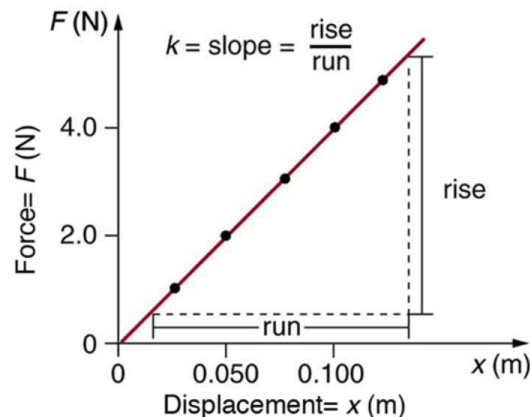
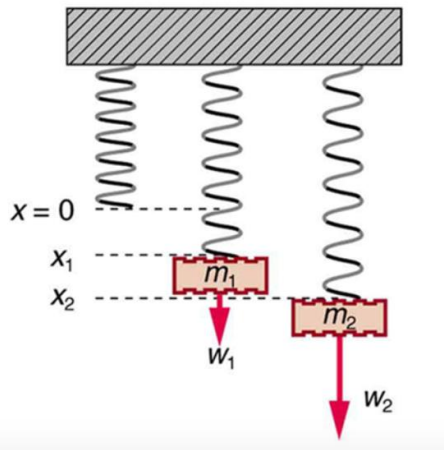
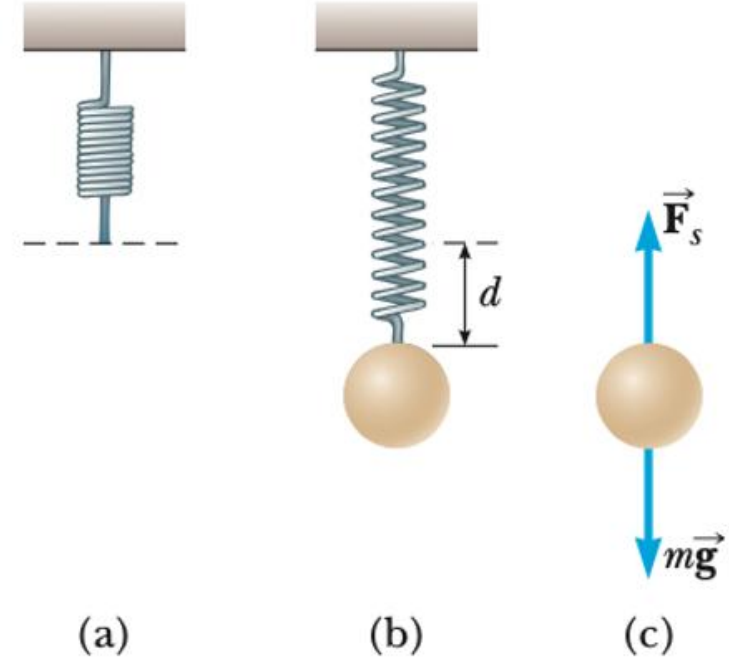
Example: Measuring the Spring Constant

A spring is hung vertically, and an object of mass m is attached to the lower end of the spring and slowly lowered a distance d to the equilibrium point. Find the value of the spring constant if the spring is displaced by 2.00 cm and the mass is 0.550 kg.

Solution:

$$\sum F_y = -F_w + F_s = -mg + kd = 0$$

$$k = \frac{mg}{d} = \frac{(0.550 \text{ kg})(9.81 \text{ ms}^2)}{0.0200 \text{ m}} = 2.7 \times 10^2 \text{ N/m}$$



m (kg)	w (N)	x (m)
0.000	0.00	0.000
0.100	0.98	0.025
0.200	1.96	0.050
0.300	2.94	0.076
0.400	3.92	0.099
0.500	4.90	0.127

This is essentially Hooke's experiment. Using a single spring, he varied the mass and recorded the equilibrium position, plotted Force vs. Displacement, found a linear relationship, and measured the slope to be the spring constant.

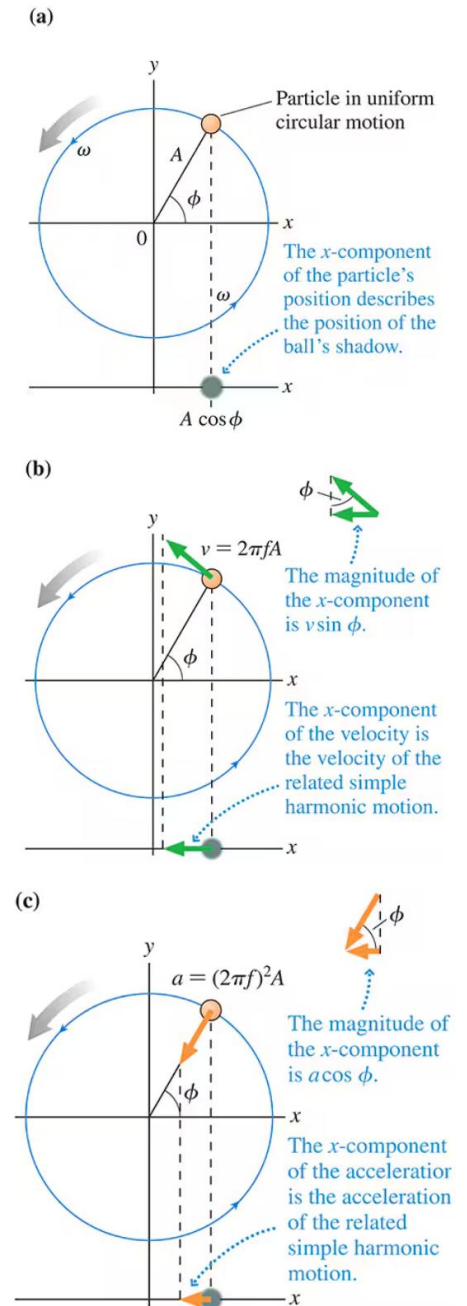
$$k = \frac{y - y_0}{x - x_0} \approx \frac{4.90 \text{ N} - 0.98 \text{ N}}{0.127 \text{ m} - 0.025 \text{ m}} = 38.43 \text{ N/m}$$

Uniform Circular Motion is Simple Harmonic Motion!

$$\begin{array}{lcl}
 x(t) = A \cos(\omega t) & \omega = 2\pi f = 2\pi/T & x(t) = x_{max} \cos(2\pi f t) \\
 v(t) = -A\omega \sin(\omega t) & \longrightarrow & v(t) = -v_{max} \sin(2\pi f t) \\
 a(t) = -A\omega^2 \cos(\omega t) & & a(t) = -a_{max} \cos(2\pi f t)
 \end{array}$$

$$\begin{array}{lcl}
 x_{max} = A \rightarrow A\omega = x_{max} \left(\frac{2\pi}{T} \right) = \frac{2\pi x_{max}}{T} = v_{max} & \text{Uniform circular motion!} & v = \frac{2\pi r}{T} \\
 A\omega^2 = x_{max} \left(\frac{2\pi}{T} \right)^2 = \frac{1}{x_{max}} \left(\frac{2\pi x_{max}}{T} \right)^2 = \frac{v_{max}^2}{x_{max}} = a_0 & \longrightarrow & a_c = \frac{v^2}{r}
 \end{array}$$

In uniform circular motion, the centripetal acceleration points toward the center. When we look at just the x-component, that inward acceleration becomes the restoring acceleration in simple harmonic motion.



Example: Measuring the sway of a tall building in the wind

The John Hancock Center in Chicago is 100 stories high. Strong winds can cause the building to sway, as is the case with all tall buildings. On particularly windy days, the top of the building oscillates with an amplitude of 40 cm ($\approx 16\text{ in}$) and a period of 7.7 s . What are the maximum speed and acceleration of the top of the building?

Solution: We assume that the oscillation of the building is simple harmonic motion with an amplitude of $A = 0.40\text{ m}$. The frequency can be computed from the period:

$$v_{\max} = \frac{2\pi A}{T} = \frac{2\pi(0.40\text{ m})}{7.7\text{ s}} = 0.33\text{ m/s}$$

$$a = \frac{v_{\max}^2}{A} = \frac{\left(\frac{2\pi A}{T}\right)^2}{A} = \left(\frac{2\pi}{T}\right)^2 A = \left(\frac{2\pi}{7.7\text{ s}}\right)^2 (0.40\text{ m}) = 0.27\text{ m/s}^2$$



Example: Sinusoidal Motion!

- a. Find the amplitude, frequency, and period of motion for an object vibrating at the end of a horizontal spring if the equation for its position as a function of time is

$$x(t) = (0.250 \text{ m}) \cos\left(\frac{\pi}{8.00} t\right)$$

- b. Find the maximum magnitude of the velocity and acceleration.
c. What are the position, velocity, and acceleration of the object after 1.00 s has elapsed?

Solution:

$$a. \quad A = 0.250 \text{ m}, \quad 2\pi f = \frac{\pi}{8.00} \rightarrow f = \frac{1}{2 \cdot 8.00} = 0.0625 \text{ Hz}, \quad T = \frac{1}{f} = \frac{1}{0.0625} \text{ s} = 16 \text{ s}$$

$$b. \quad v_{\max} = \frac{2\pi A}{T} = \frac{2\pi(0.250 \text{ m})}{16 \text{ s}} = 0.098 \text{ m/s} \quad a_{\max} = \frac{v_{\max}^2}{A} = \frac{(0.098 \text{ m/s})^2}{0.250 \text{ m}} = 0.038 \text{ m/s}^2$$

$$c. \quad x(t = 1.00 \text{ s}) = (0.250 \text{ m}) \cos\left(\frac{\pi}{8.00} (1.00 \text{ s})\right) = 0.231 \text{ m}$$

$$v(t = 1.00 \text{ s}) = -(0.098 \text{ m/s}) \sin\left(\frac{\pi}{8.00} (1.00 \text{ s})\right) = -0.038 \text{ m/s}$$

$$a(t = 1.00 \text{ s}) = -(0.038 \text{ m/s}^2) \cos\left(\frac{\pi}{8.00} (1.00 \text{ s})\right) = -0.035 \text{ m/s}^2$$

Energy in Simple Harmonic Motion 1/2

When a constant force is used to displace something, the work done is,

$$W = Fx$$

But a linear restoring force takes the form,

$$F(x) = kx$$

The force itself is a function of displacement and therefore is not constant!

$$W = F_{avg}x = \frac{F_f - F_0}{2}x = \frac{kx - 0}{2}x = \frac{1}{2}kx^2$$

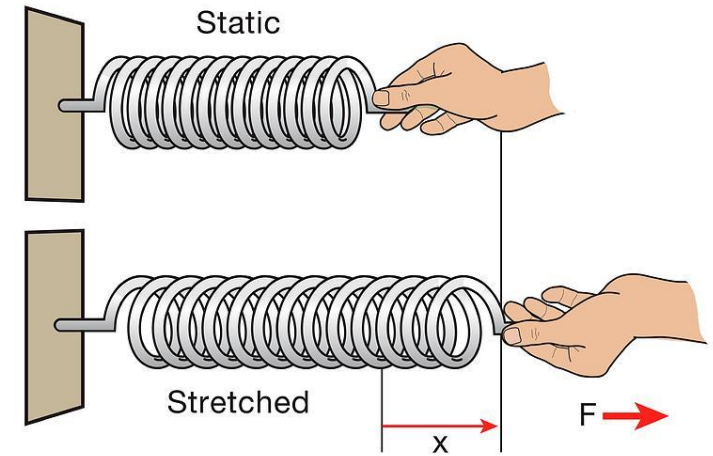
The work is stored as potential energy in the spring

$$U_s = \frac{1}{2}kx^2$$

This is called elastic potential energy of a spring displaced a distance x from equilibrium.

Notice that if the maximum distance that we pull it back is $x = A$, the maximum potential energy stored in the spring is,

$$U_{max} = \frac{1}{2}kA^2$$



In calculus,

$$\begin{aligned} W &= \int_0^x F(x') dx' \\ &= \int_0^x kx' dx' = \frac{1}{2}kx^2 \end{aligned}$$

Energy in Simple Harmonic Motion 2/2

Ignoring friction, we have no energy loss, so the conservation of energy for this system,

$$E_0 = \frac{1}{2}mv_0^2 + \frac{1}{2}kA^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$$

The system starts from rest $v_0 = 0$, solving for v_f :

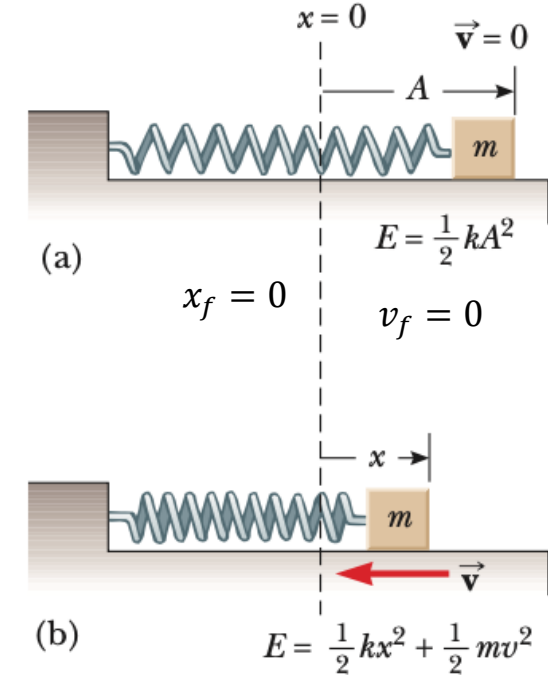
$$v_f = \sqrt{\frac{k}{m}(A^2 - x_f^2)}$$

When the block passes back through the equilibrium position at $x_f = 0$,

$$v_f = \sqrt{\frac{k}{m}A^2} = A\sqrt{\frac{k}{m}}$$

It reaches its maximum velocity because all of that stored potential energy in the spring has now been transformed into kinetic energy,

$$U_{max} = \frac{1}{2}kA^2 \quad \rightarrow \quad v_{max} = A\sqrt{\frac{k}{m}}$$



Recall earlier when we analyzed via uniform circular motion,

$$v_{max} = \frac{2\pi A}{T} = 2\pi f A$$

$$2\pi f A = A\sqrt{\frac{k}{m}} \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$

Example: Total energy of a Spring.

A 0.500 kg object connected to a light spring with a spring constant of 20.0 N/m oscillates on a frictionless horizontal surface.

- Calculate the total energy of the system.
- Calculate the maximum speed of the object if the amplitude of motion is 3.00 cm.
- What is the velocity of the object when the displacement is 2.00 cm?
- Compute the kinetic and potential energies of the system when the displacement is 2.00 cm.

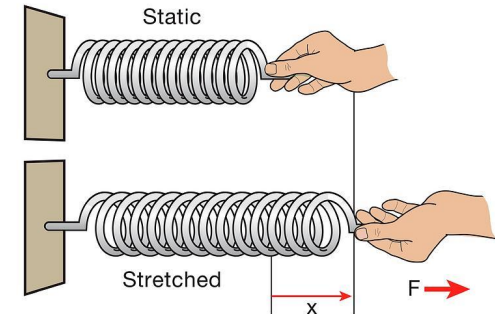
Solution:

$$a. \quad E = KE + PE = 0 + \frac{1}{2}kA^2 = \frac{1}{2}\left(20.0\frac{\text{N}}{\text{m}}\right)(3.00 \times 10^{-2} \text{ m})^2 = 9.00 \times 10^{-3} \text{ J}$$

$$b. \quad E_0 = E_f \rightarrow (KE + PE)_0 = (KE + PE)_f \rightarrow 0 + \frac{1}{2}kA^2 = \frac{1}{2}mv_{\text{max}}^2 + 0 \rightarrow v_{\text{max}}^2 = \frac{kA^2}{m} \rightarrow v_{\text{max}} = A\sqrt{\frac{k}{m}} = (0.03 \text{ m})\sqrt{\frac{20.0 \text{ Nm}^{-1}}{0.500 \text{ kg}}} \approx 0.190 \text{ m/s}$$

$$c. \quad v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\sqrt{\frac{20 \text{ Nm}^{-1}}{0.500 \text{ kg}}[(0.03 \text{ m})^2 - (0.02 \text{ m})^2]} = \pm 0.141 \text{ m/s}$$

$$d. \quad KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.500 \text{ kg})(0.141 \text{ ms}^{-1})^2 = 4.97 \times 10^{-3} \text{ J} \quad PE = \frac{1}{2}kx^2 = \frac{1}{2}(20.0 \text{ Nm}^{-1})(2.00 \times 10^{-2} \text{ m})^2 = 4.00 \times 10^{-3} \text{ J}$$



The Pendulum

When displaced from equilibrium, the mass oscillates around the equilibrium position. The restoring force is the force responsible for bringing the mass back to its equilibrium position. Notice,

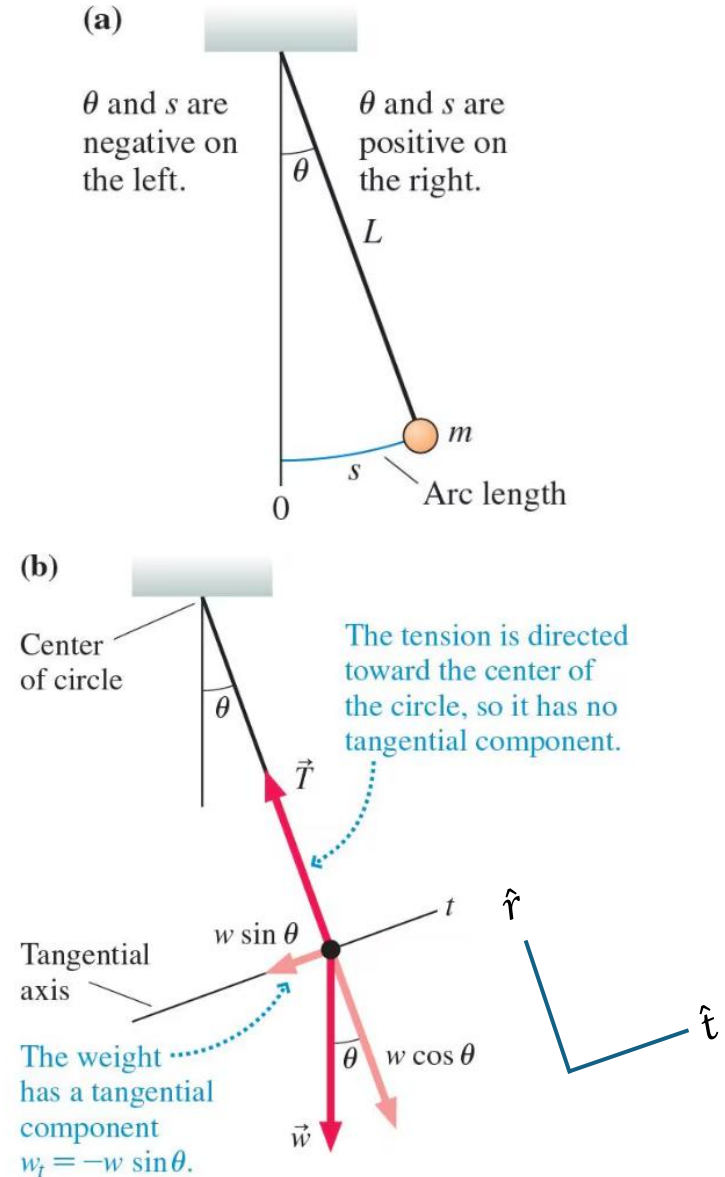
$$\sum F_r = F_{T_r} - w_r = ma_r = 0 \rightarrow F_T = mg \cos \theta$$

But this only tells us that T is the radial component of the weight, but there is not motion in \hat{r} . Summing the forces in the tangential direction,

$$\sum F_t = -w_t = -mg \sin \theta$$

This is the component of the net force responsible for bringing the mass back toward its equilibrium position. If we use the small angle approximation $\sin \theta \approx \theta$ which is valid for $\theta < 10^\circ$, and recognize that the arc length is $s = r\theta = L\theta$,

$$\sum F_t = (F_{net})_t = -mg \sin \theta \approx -mg\theta = -\left(\frac{mg}{L}\right)s$$



Pendulum Motion

For a pendulum of length L displaced by an arc length s for small angles θ , the tangential restoring force is,

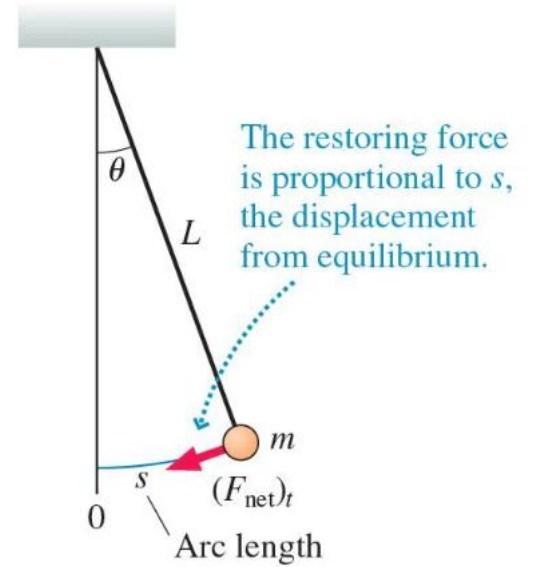
$$(F_{net})_t = -\frac{mg}{L}s$$

Recall for a mass spring system, $F_x = -kx$, we found the frequency,

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$(F_{net})_t$ has a similar form to $F_x = -kx$, if $k = mg/L$,

$$f = \frac{1}{2\pi} \sqrt{\frac{mg}{mL}} = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \quad T = \frac{1}{f} = 2\pi \sqrt{\frac{L}{g}}$$



For the pendulum system described the frequency and period:

- Are independent of amplitude – just like a mass-spring system.
- Are independent of mass – unlike a mass-spring system.

Example: Lunar Pendulum

The free-fall acceleration on the moon is 1.62 ms^{-2} . What is the length of a pendulum whose period on the moon matches the period of 2.00 m long pendulum on the Earth?

Solution:

For a simple pendulum, the period ,

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$T_M = 2\pi \sqrt{\frac{L_M}{g_M}} = 2\pi \sqrt{\frac{L_E}{g_E}} = T_E \quad \rightarrow \quad L_M = L_E \frac{g_M}{g_E} = (2.00 \text{ m}) \frac{1.62}{9.81} = 0.33 \text{ m}$$

Example: Equation of Motion for a Simple Pendulum

A simple pendulum of length L and mass m is pulled to a small angle θ_0 (measured from the vertical) and released from rest at $t = 0$. The angular position is given by,

$$\theta(t) = \theta_0 \cos(\omega t)$$

- Write the above equation in terms of $s(t)$, $v(t)$, and $a(t)$ in terms of s_0 , L , g , t .
- Find the max kinetic energy in terms of m , g , L , and θ_0 . Where does it occur?
- Find the max potential energy in terms of m , g , L , and θ_0 . Where does it occur?

Solution:

a. Using $s = L\theta$, and $f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$, and $\omega = 2\pi f$

$$\frac{s(t)}{L} = \frac{s_0}{L} \cos\left(\sqrt{\frac{g}{L}} t\right) \rightarrow s(t) = s_0 \cos\left(\sqrt{\frac{g}{L}} t\right)$$

$$v(t) = -s_0 \sqrt{\frac{g}{L}} \sin\left(\sqrt{\frac{g}{L}} t\right) \quad a(t) = -s_0 \left(\sqrt{\frac{g}{L}}\right)^2 \cos\left(\sqrt{\frac{g}{L}} t\right)$$

b. $KE_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2} m \left(s_0 \sqrt{\frac{g}{L}}\right)^2 = \frac{1}{2} m (L\theta_0)^2 \frac{g}{L} = \frac{1}{2} mgL\theta_0^2$

Occurs at the bottom of the swing.

c. $U_{max} = \frac{1}{2} k x_{max}^2 = \frac{1}{2} \left(\frac{mg}{L}\right) s_0^2 = \frac{1}{2} \frac{mg}{L} (L\theta_0)^2 = \frac{1}{2} mgL\theta_0^2$

Occurs at the top of the swing.

Damped Oscillations

In the real world the oscillation of a pendulum would slowly decrease in amplitude due to air resistance – an oscillation that runs down and stops is called a damped oscillation. A result from calculus shows us,

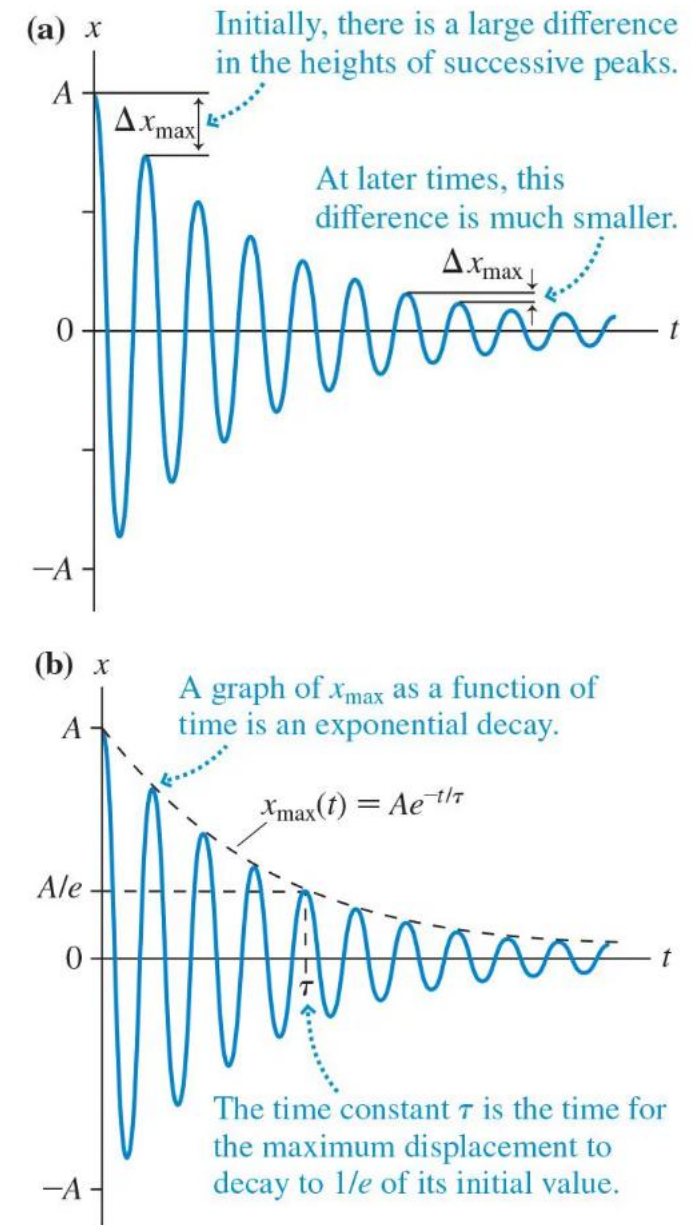
$$x_{max}(t) = Ae^{-t/\tau}$$

Where $e \approx 2.718$ is the base of the natural log and A is the initial amplitude (displacement). The constant τ is called the time constant. When $t = \tau$, the maximum displacement x_{max} has decreased to,

$$x_{max}(t = \tau) = Ae^{-1} = \frac{A}{e} \approx 0.37A$$

The oscillation amplitude has decreased to about 37% of its initial value.

An oscillation that decays quickly has a smaller τ and one that decays slowly has a larger τ .



Example: Finding a clock's decay time

The pendulum in a grandfather clock has a period of 1.00 s. If the clock's driving spring is allowed to run down, damping due to friction will cause the pendulum to slow to a stop. If the time constant for this decay is 300s, how long will it take for the pendulum's swing to be reduced to half its initial amplitude?

Solution:

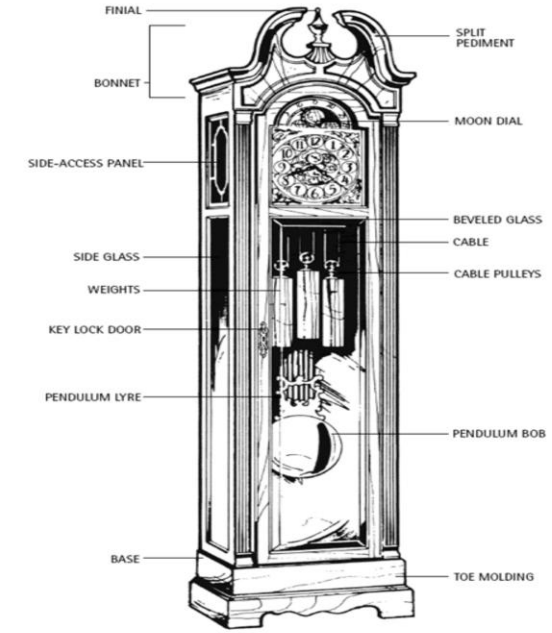
$$x_{max}(t) = Ae^{-t/\tau} \rightarrow \theta_{max}(t) = \theta_0 e^{-t/\tau} = \frac{1}{2} \theta_0$$

$$e^{-t/\tau} = \frac{1}{2}$$

$$\ln(e^{-t/\tau}) = \ln\left(\frac{1}{2}\right)$$

$$-\frac{t}{\tau} \ln e = -\ln(2)$$

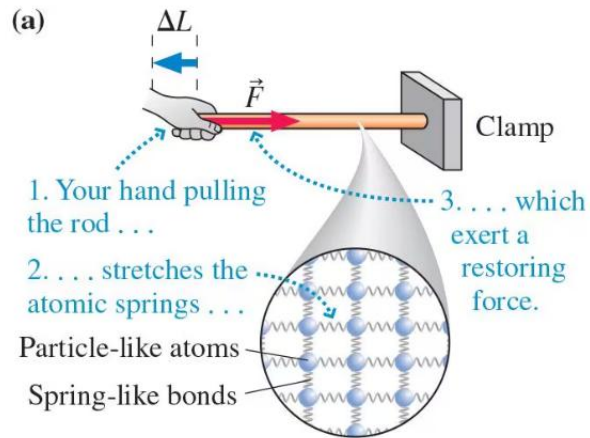
$$t = \tau \ln 2 = (300 \text{ s}) \ln 2 = 208 \text{ s}$$



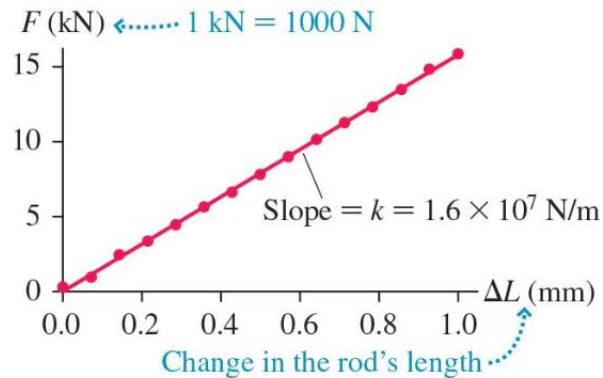
It will take 208 s or about 3.5 min for the oscillations to decay by half after the spring has run down.

$t < \tau$, which makes sense. The time constant is the time for the amplitude to decay to 37% of its initial value; we are looking for the time to decay to 50% of its initial value, which should be a shorter time. The time to decay to $\frac{1}{2}$ of the initial value, $t = \tau \ln 2$ could be called the half-life. We will see this again next quarter.

Stretching and Compressing Materials



(b) Data for the stretch of a 1.0-m-long, 1.0-cm-diameter steel rod



Consider a steel rod, we can model as made up of spring-like bonds between atoms in steel and while stiff can still be stretched or compressed – which means it has a “spring constant”.

We expect this constant to be a function of several factors:

- Cross-sectional area, A : thick rod is harder to stretch than a thin one.
- Length, L : Long rod easier to stretch than a short rod.
- The material the rod is made from. Steel rod vs rubber rod.

Experimental result,

$$k = \frac{YA}{L}$$

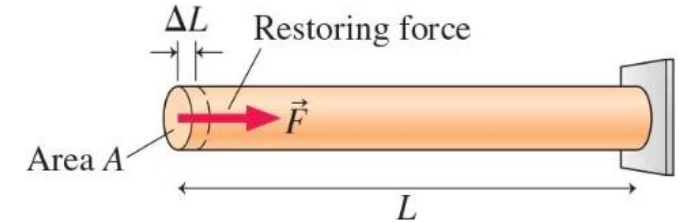
Where Y is called Young's modulus and is a property of the material. Recall that a linear restoring force, like a mass-spring system, follows $F = k\Delta x$,

$$F = \frac{YA}{L} \Delta L$$

The ratio of force to cross-section area is called **stress**.

$$\frac{F}{A} = Y \left(\frac{\Delta L}{L} \right)$$

The ratio of the change in length to the original length is called **strain**.



Material	Young's modulus (10^{10} N/m^2)
Cast iron	20
Steel	20
Silicon	13
Copper	11
Aluminum	7
Glass	7
Concrete	3
Wood (Douglas Fir)	1

Example: Finding the Stretch of a Cable

A Foucault pendulum consists of a 120 kg steel ball that swings at the end of a 6.0 m long steel cable. The cable has a diameter of 2.5 mm. When the ball was first hung from the cable, by how much did the cable stretch?

Solution: Young's modulus for steel is $Y = 20 \times 10^{10} \text{ N/m}^2$. The cross-sectional area of the cable,

$$A = \pi r^2 = \pi(0.00125 \text{ m})^2 = 4.91 \times 10^{-6} \text{ m}^2$$

Now rearrange the equation for the cable's restoring force,

$$F = Y \frac{\Delta L}{L} \rightarrow \Delta L = \frac{LF}{AY} = \frac{L(mg)}{AY} = \frac{(6.0 \text{ m})(120 \text{ kg})(9.8 \text{ m/s}^2)}{(4.91 \times 10^{-6} \text{ m}^2)(20 \times 10^{20} \text{ N/m}^2)}$$
$$= 0.0072 \text{ m} = 7.2 \text{ mm}$$



Material	Young's modulus (10^{10} N/m^2)
Cast iron	20
Steel	20
Silicon	13
Copper	11
Aluminum	7
Glass	7
Concrete	3
Wood (Douglas Fir)	1



Physicist Spotlight: Robert Hooke

Robert Hooke (1635–1703) was a renowned 17th-century English scientist, inventor, and polymath, known for his discovery of **Hooke's Law** of elasticity and pioneering work in microscopy, where he coined the term "cell." As Curator of Experiments for the Royal Society, he made significant contributions to mechanics, astronomy, and architecture, notably aiding in London's post-fire reconstruction. Hooke had a famously bitter rivalry with Isaac Newton, particularly over the inverse-square law of gravity and optics, which overshadowed his legacy. Despite this, Hooke's wide-ranging contributions cemented him as one of the era's most influential scientists.



From Pendulum to Particles: Why SHM Is Everywhere 1/2

We learned that

$$F_x = -kx \rightarrow F_x + kx = 0 \rightarrow ma_x + kx = 0$$

Another way we can write this by using more modern notation, $a_x = \ddot{x}$, and we already know its solution, $x(t)$,

$$m\ddot{x} + kx = 0, \quad x(t) = A \cos(\omega t + \varphi), \quad \omega = \sqrt{\frac{k}{m}}$$

This is what we call an “equation of motion” and its solution. In classical physics we can know exactly where something is and how fast it’s moving. That means x and momentum p are just numbers you can measure simultaneously. Example: A pendulum bob at $x = 2 \text{ cm}$, $v = 0.5 \text{ m/s}$.

But in the quantum world particles behave like waves:

- You can describe a wave’s position or its momentum – but not exactly at once.
- This is the Heisenberg Uncertainty Principle: $\Delta x \Delta p \geq \frac{\hbar}{2}$.

So “position” and “momentum” are no longer fixed numbers – they’re linked by wave behavior. To describe this mathematically, we can’t treat x and p as simple variables anymore. We must use operators!

From Pendulum to Particles: Why SHM Is Everywhere 2/2

$$m\ddot{x} + kx = 0, \quad x(t) = A \cos(\omega t + \varphi), \quad \omega = \sqrt{\frac{k}{m}}$$

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 \rightarrow H(x, p) = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

When we quantize something, we replace the x and y with operators. In the Hamiltonian, H , it becomes the Hamiltonian operator,

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2$$

When this operator acts on a wave function representing a particle, it yields the allowed energy states,

$$\hat{H}\psi = E\psi$$

This is called the time-independent Schrodinger equation. We now have our **quantum harmonic oscillator**. Now, imagine not one oscillator - but a whole **chain** of them, each connected to its neighbors. The result is a **wave of excitations** that can travel along the chain.

Quantum Field Theory takes that same idea and **extends it to all of space**:

- Every **point in space** is treated like a tiny harmonic oscillator.
- The “field” is a **collection of infinitely many** such oscillators.
- When one of them is excited, that excitation is what we call a **particle**.

In QFT, particles are just vibrations (quantum excitations) of underlying fields - exactly like how a guitar string's vibration is made of many tiny harmonic oscillations.

