

Lecture Notes: Electrostatics

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1 About This Lecture

These notes cover the core ideas typically found in a standard algebra-based physics text. The organization here differs intentionally from the textbook.

The goal is to present the material in a way that emphasizes physical intuition, conceptual coherence, and problem-solving structure. Rather than following the chapter sequence exactly, the topics are arranged to highlight the underlying principles that unify electrostatics.

2 Introduction

What happens when you rub a balloon on your hair? Your hair starts to rise, reaching toward the balloon. If you bring the balloon near small pieces of paper, they jump up toward it. Somehow, the balloon has gained an invisible property that allows it to pull on other objects. But what is this property?

This mysterious effect is caused by electric charge—a fundamental property of matter. Objects can gain or lose charge, which allows them to exert forces on each other, even when they're not touching. But charge isn't just some classroom trick—it's the reason why atoms hold together, why lightning strikes, and why circuits power our devices.

By understanding charge, we can answer important questions:

- Why do some objects attract while others repel?
- What does it mean for something to be "neutral"?
- How does charge move from one object to another?

Electrostatics is the study of electric charges at rest and the forces they exert on one another. It explains how charged objects interact without direct contact and how these interactions shape the behavior of materials and systems. Unlike moving charges, which create electric currents and magnetic fields, electrostatics deals with stationary charges and the electric fields they produce.

3 Electric Charge

Electric charge is a fundamental property of matter that determines how objects interact through electrostatic forces. While we often treat charge as a simple number, it represents a deeper physical reality: the presence or absence of elementary charges carried by particles like electrons and protons.

At the smallest level, charge is quantized, meaning it always comes in integer multiples of a fundamental unit of charge:

$$q = ne$$

where, q is the charge of an object, $e = 1.602 \times 10^{-19}$ C, and n is an integer representing the number of excess or missing electrons. The unit, the coulomb (C), is defined as the charge of approximately 6.242×10^{18} electrons.

A **point charge** is an idealized model used in physics to represent an electric charge that is concentrated at a single point in space. It is a useful approximation that simplifies calculations involving electric forces and fields while still capturing the essential behavior of charge interactions. In reality, all charged objects have some physical size, but in many cases, their dimensions are small enough that they can be treated as if all their charge is concentrated at a single point. This makes mathematical analysis easier, particularly when applying Coulomb's Law and electric field equations.

Electric charge can be transferred between objects in different ways. Two common methods of charging an object are **charging by conduction** and **charging by induction**.

3.1 Conductors and Insulators

Materials can be classified into two broad categories based on how they allow electric charge to move: **conductors** and **insulators**.

Conductors are materials that allow electric charge to move freely. In a conductor, electrons are loosely bound to atoms and can move throughout the material in response to an electric field. Common examples include:

- Metals such as copper, silver, and aluminum.
- Saltwater and other ionic solutions.
- Plasma, which consists of freely moving charged particles.

A key property of conductors is that when they reach **electrostatic equilibrium**, any excess charge resides on the surface, and the electric field inside the conductor is zero.

Insulators, on the other hand, do not allow charge to move freely. In these materials, electrons are tightly bound to atoms and cannot easily move. Common examples include:

- Rubber and plastic.
- Glass and ceramics.
- Pure (distilled) water.

In an insulator, excess charge remains localized rather than spreading across the surface, and electric fields can exist inside the material.

Semiconductors are a third category of materials that have properties between those of conductors and insulators. Their ability to conduct electricity can be controlled, making them essential for electronic devices such as transistors and diodes.

Understanding the behavior of conductors and insulators is crucial for applications ranging from electrical wiring to electrostatic shielding.

3.2 Charging by Conduction

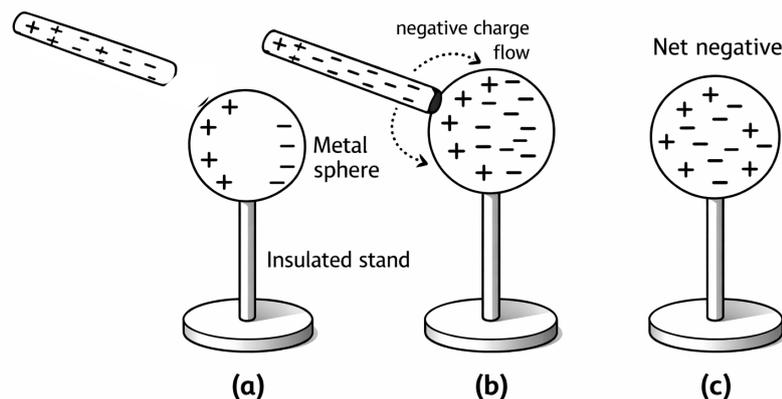


Figure 1: Charging by Conduction Process: (a) A negatively charged rod is brought into contact with a neutral metal sphere mounted on an insulated stand. The sphere initially contains equal amounts of positive and negative charge. (b) When the rod touches the sphere, negative charge (electrons) flows between the rod and the sphere until they reach electrostatic equilibrium. (c) After the rod is removed, the sphere is left with an excess of negative

Charging by conduction, as shown in Fig. 1, occurs when a charged object physically touches a neutral conductor, allowing negative charge to flow between them. The neutral object ends up with the **same type of charge** as the charged object. The process:

1. A charged object is brought into direct contact with a neutral conductor.
2. Electrons transfer between the objects until they reach electrostatic equilibrium.
3. After separation, the previously neutral object is left with the same type of charge as the charged object.

3.3 Charging by Induction

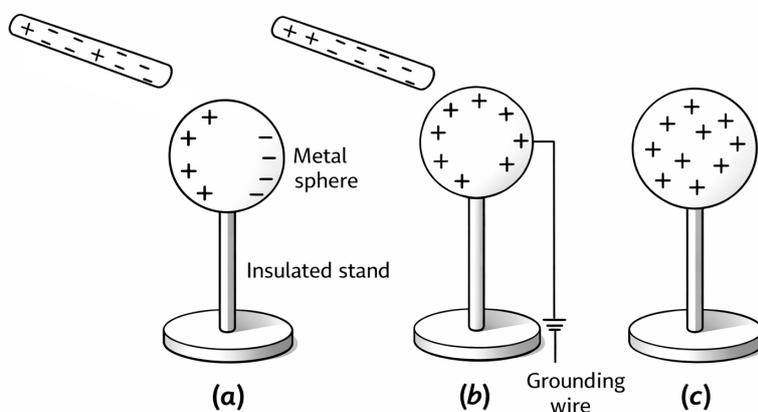


Figure 2: Charging by Induction Process: (a) A negatively charged rod is brought near a neutral metal sphere, causing charge separation. (b) The sphere is connected to the ground, allowing negative charge to flow away. (c) The ground connection is removed while the rod is still present. (d) The sphere is left with an induced positive charge after the rod is taken away.

Induction charging allows an object to be charged **without direct contact**. Instead of transferring charge, a nearby charged object causes a redistribution of charge in the neutral object. The process as shown in Fig. 2:

1. A charged object is brought close to (but does not touch) a neutral conductor.
2. The presence of the charged object causes charges in the neutral object to rearrange—like charges repel, and opposite charges attract.
3. If the conductor is grounded, some of the like charges move to the ground.
4. The ground connection is removed, and then the charged object is taken away, leaving the neutral object with an opposite charge.

3.4 Conservation of Charge

One of the fundamental principles of physics is the **law of charge conservation**, which states that the total electric charge in an isolated system remains constant. This means that charge can be transferred from one object to another, but it cannot be created or destroyed.

Mathematically, if a system starts with an initial charge Q_{initial} and undergoes any physical processes, the total charge afterward must still be the same:

$$Q_{\text{final}} = Q_{\text{initial}}.$$

This principle applies in all physical processes, including:

- Charging an object by conduction or induction—charge moves, but the total remains constant.
- Chemical reactions, such as those in batteries, where charge is redistributed but not created or lost.
- Electric circuits, where charge flows through components but never accumulates or disappears.

An important consequence of charge conservation is that any charge introduced into a conductor redistributes itself while maintaining the total charge. For example, in a conducting shell, if charge accumulates on one surface, an equal and opposite charge must appear elsewhere to satisfy charge conservation.

Charge conservation is a fundamental law in classical and modern physics and has never been observed to be violated.

Note: Here we use Q rather than q because the latter is typically reserved for a point charge, whereas the former can explain charged objects and collections of point charges, in general.

3.5 Van de Graaff Generator

A Van de Graaff generator is a device that produces high-voltage static electricity, often seen in science demonstrations where a person's hair stands on end. It works by continuously transferring charge to a large metal sphere, creating strong electric fields and noticeable electrostatic effects. When a person touches the dome, they receive charge via conduction. However it also charges by induction. Nearby objects can be charged without direct contact and paper or aluminum reacts to it without touching.

4 Coulomb's Law

4.1 Two Point Charges

The electrostatic force F between two point charges q_1 and q_2 separated by a distance r is given by:

$$F = k_e \frac{|q_1 q_2|}{r^2} \quad (1)$$

where:

- F is the magnitude of the electrostatic force (in newtons, N),
- q_1 and q_2 are the magnitudes of the two charges (in coulombs, C),
- r is the distance between the charges (in meters, m),
- $k_e = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ is Coulomb's constant.

If q_1 and q_2 have the same sign (both positive or both negative), the force is **repulsive**. If q_1 and q_2 have opposite signs, the force is **attractive**. The force acts along the straight line joining the two charges.

A note on constants: Two constants that can be used interchangeably are Coulomb's constant k and the permittivity of free space ϵ_0 . They are related by the equation:

$$k = \frac{1}{4\pi\epsilon_0}$$

. Given k above, find the value of ϵ_0 .

4.2 Example Problem 1

Problem: Two point charges are placed in vacuum as shown in Fig. 4.2. Charge $q_1 = +3.0\mu\text{C}$ is positioned at $x = 0$, and charge $q_2 = -2.0\mu\text{C}$ is placed at $x = 0.5 \text{ m}$. Determine the magnitude and direction of the electrostatic force exerted by q_1 on q_2 .

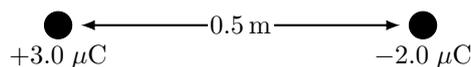


Figure 3: Two point charges separated by 0.5 m.

Solution

Using Coulomb's Law:

$$F = k_e \frac{|q_1 q_2|}{r^2} \quad (2)$$

where:

- $k_e = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$ is Coulomb's constant,
- $q_1 = +3.0 \times 10^{-6} \text{ C}$,
- $q_2 = -2.0 \times 10^{-6} \text{ C}$,
- $r = 0.5 \text{ m}$.

Step 1: Calculate the Magnitude of the Force

$$\begin{aligned}
 F &= (8.99 \times 10^9) \frac{(3.0 \times 10^{-6})(2.0 \times 10^{-6})}{(0.5)^2} \\
 &= (8.99 \times 10^9) \frac{6.0 \times 10^{-12}}{0.25} \\
 &= 0.216 \text{ N}
 \end{aligned}$$

Step 2: Determine the Direction

Since q_1 is positive and q_2 is negative, the force between them is attractive. This means:

- q_1 pulls q_2 to the left.
- q_2 pulls q_1 to the right.

Final Answer: The magnitude of the electrostatic force is 0.216 N, directed toward each other.

4.3 Net Force on a Point Charge from Multiple Charges

If a charge q experiences forces from multiple charges, the total force is found by vector addition:

$$\mathbf{F}_{\text{net}} = \sum \mathbf{F}_i$$

where \mathbf{F}_i represents the force from each individual charge. Because forces are vectors, we must consider both magnitude and direction.

Notation: In these notes, bold face type such as \mathbf{V} , notates a vector and is equivalent to \vec{V} . The magnitude of a vector will be denoted without bold face or an arrow, such as V . Often, outside of these notes, you will see the magnitude of a vector also written as $|\vec{V}|$ or $|\mathbf{V}|$.

4.4 Example: Three Charges in a Line

Consider three point charges arranged along a straight line as shown in Fig. 4:

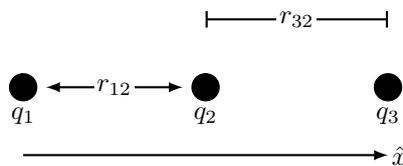


Figure 4: Three point charges arranged on a line.

The net force on q_2 is the sum of forces due to q_1 and q_3 . Each force is calculated using Coulomb's Law and then summed as vectors.

Force from q_1 on q_2

The force on q_2 due to q_1 is:

$$F_{12} = k_e \frac{|q_1 q_2|}{r_{12}^2}$$

where r_{12} is the distance between q_1 and q_2 . The direction of this force depends on the signs of q_1 and q_2 .

Force from q_3 on q_2

Similarly, the force on q_2 due to q_3 is:

$$F_{32} = k_e \frac{|q_3 q_2|}{r_{32}^2}$$

where r_{32} is the distance between q_2 and q_3 . Again, the direction depends on the charge signs.

Net Force on q_2

Since all forces lie along the same line, we can assign signs based on direction (e.g., rightward as positive and leftward as negative) and write:

$$F_{\text{net},2} = F_{12} + F_{32}$$

This equation provides the net force on q_2 , where the direction depends on the relative magnitudes of F_{12} and F_{32} .

- If q_1 and q_3 are equal in magnitude and equidistant from q_2 , the net force on q_2 will be zero if both forces are equal and opposite.
- If one force is stronger than the other, the net force will point in that direction.
- If the charges are not symmetrically arranged, a net force will remain, and its direction will depend on the relative charge magnitudes and distances.

4.5 Vector Components in 2D

Recall that forces are vectors, which means they have both magnitude and directions. Consider Fig. 5. We know that q_1 , which is at the origin, is positive, and q_2 which is in quadrant I, is negative. Therefore the force on q_1 due to q_2 can be represented as a vector, \vec{F}_{21} , which points from q_1 to q_2 . This force vector can be broken down into its components along the x and y axes. The x -component of the force is $F_{21} \cos \theta$, and the y -component is $F_{21} \sin \theta$, where θ is the angle between the force vector and the positive x -axis. The magnitude of F_{21} can be related to its components using the Pythagorean theorem:

$$F_{21} = \sqrt{(F_{21} \cos \theta)^2 + (F_{21} \sin \theta)^2} = F_{21} \sqrt{\cos^2 \theta + \sin^2 \theta} = F_{21}$$

The above seems silly, but is to remind you of how vectors work, and will be useful later when we have more than 2 charges. We can relate the angle, θ , to the components of the force using the inverse tangent function:

$$\theta = \tan^{-1} \left(\frac{F_{21} \cos \theta}{F_{21} \sin \theta} \right) = \tan^{-1} \left(\frac{\cos \theta}{\sin \theta} \right) = \tan^{-1}(\tan \theta) = \theta$$

Again, obviously, but in the next example when we have three charges in 2D, this will be useful to find the direction of the net force.

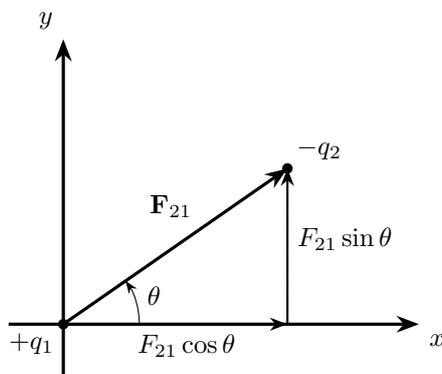


Figure 5: A point charge is at the origin, the force vector represents the force on this charge due to another located in quadrant I.

What would our diagram look like if q_2 were also positive? Fig. 6 shows just this scenario. θ in quadrant III is between 180° and 270° , so $\cos \theta$ and $\sin \theta$ are both negative, which means the components of the force are negative, which is consistent with what we see in the diagram.

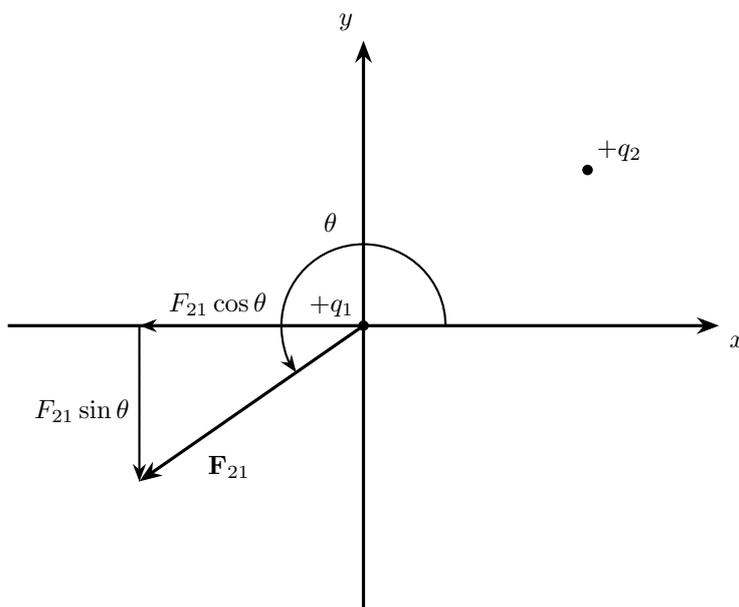


Figure 6: If both charges are positive, the force on q_1 due to q_2 is repulsive and points into quadrant III.

4.6 Example: Net Force on a Charge in 2D

Consider three charges arranged in a 2D plane as shown in Fig 7. A charge q_1 is placed at the origin, and it experiences forces from two other charges, q_2 and q_3 , located at known positions. q_1 is positive, and q_2 and q_3 are negative.

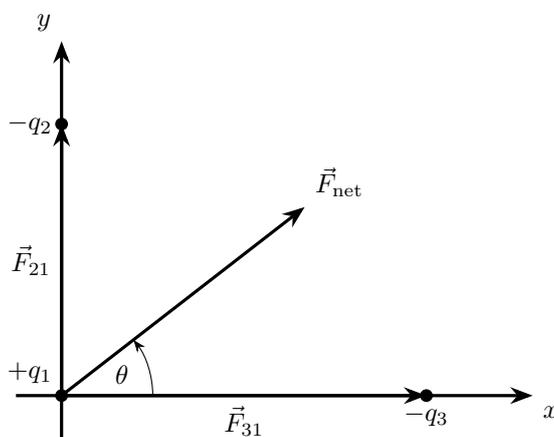


Figure 7: A point charge is at the origin, with two negative charges at distances away, along the x and y axes.

Step 1: Compute Individual Forces

Using Coulomb's Law, the magnitude of each force is:

$$F_{21} = k_e \frac{|q_2 q_1|}{r_{21}^2}, \quad F_{31} = k_e \frac{|q_3 q_1|}{r_{31}^2}$$

where: - r_{21} is the distance between q_1 and q_2 , - r_{31} is the distance between q_1 and q_3 .

Since q_3 is along the $+x$ -axis, \mathbf{F}_{31} has only an x -component:

$$\mathbf{F}_{31} = k_e \frac{|q_2 q_1|}{r_{21}^2} \hat{x}$$

Similarly, q_2 is along the $+y$ -axis, so \mathbf{F}_{21} has only a y -component:

$$\mathbf{F}_{21} = k_e \frac{|q_2 q_1|}{r_{21}^2} \hat{y}$$

Step 2: Compute Net Force Components

The total force components are:

$$F_{\text{net},x} = F_{21,x} + F_{31,x} = 0 + k_e \frac{|q_3 q_1|}{r_{31}^2} = k_e \frac{|q_3 q_1|}{r_{31}^2}$$

$$F_{\text{net},y} = F_{21,y} + F_{31,y} = k_e \frac{|q_2 q_1|}{r_{21}^2} + 0 = k_e \frac{|q_2 q_1|}{r_{21}^2}$$

Step 3: Compute Magnitude and Direction

The magnitude of the net force is:

$$F_{\text{net}} = \sqrt{F_{\text{net},x}^2 + F_{\text{net},y}^2} = \sqrt{\left(k_e \frac{|q_3 q_1|}{r_{31}^2}\right)^2 + \left(k_e \frac{|q_2 q_1|}{r_{21}^2}\right)^2} = k_e |q_1| \sqrt{\left(\frac{|q_3|}{r_{31}}\right)^2 + \left(\frac{|q_2|}{r_{21}}\right)^2}$$

The direction is given by:

$$\theta = \tan^{-1} \left(\frac{F_{\text{net},y}}{F_{\text{net},x}} \right) = \tan^{-1} \left(k_e \frac{|q_2 q_1|}{r_{21}^2} \cdot \frac{r_{31}^2}{k_e |q_3 q_1|} \right) = \tan^{-1} \left[\frac{|q_2|}{|q_3|} \left(\frac{r_{31}}{r_{21}} \right)^2 \right]$$

Step 4: Interpret the Result

- If q_2 and q_3 are equal and at the same distance from q_1 , the net force will be at a 45-degree angle.
- If q_3 is stronger than q_2 , and at the same distance, the net force on q_1 will lean more toward the x -axis.
- If q_2 is stronger than q_3 , and at the same distance, the net force on q_1 will lean more toward the y -axis.

This example demonstrates how to use vector components to determine the net force on a charge when multiple forces act in different directions.

Validate the Interpretation

- Let $q_2 = q_3 = -q$ and $r_{21} = r_{31}$. Is $\theta = 45^\circ$?
- Let $q_3 = -3 \mu C$ and $q_2 = -1 \mu C$ and $q_1 = 0.5 \mu C$, and $r_{21} = r_{31}$, find the angle θ . Is it less than 45° ?
- Let $q_2 = -3 \mu C$ and $q_3 = -1 \mu C$ and $q_1 = 0.5 \mu C$, and $r_{21} = r_{31}$, find the angle θ . Is it greater than 45° ?

5 The Electric Field

5.1 What is a Field?

To a physicist, a field is a way to describe how a force exists in space, even before anything experiences it. Think of it as an invisible influence that fills a region and can exert a force on objects placed within it. Instead of thinking about forces acting only when two objects directly interact, a field allows us to think about how one object—like a planet or a charged particle—creates a "presence" that extends outward, affecting other objects that come near.

Both the gravitational field and the electric field follow this idea: they describe how forces are "waiting" in space for something to enter and feel them.

- Gravitational Field: Imagine the Earth pulling objects toward it. You don't have to touch the Earth to feel this force – it's there, affecting everything around it, pulling apples off trees and keeping the Moon in orbit. The gravitational field exists all around the planet, and anything with mass feels its influence.
- Electric Field: Now think about a charged balloon that can pull tiny pieces of paper toward it. The paper doesn't have to touch the balloon to feel the force; the charge on the balloon creates an electric field in the space around it, and that field influences the paper.

If you think of the gravitational field like a gentle "slope" in space where objects naturally roll toward a massive object, then an electric field is more like an invisible "wind" that pushes or pulls charges in a certain direction, depending on whether they are positive or negative.

Both fields describe how a force is present everywhere, even when nothing is currently feeling it. The difference is that gravity only pulls, while the electric field can push or pull depending on the type of charge involved.

5.2 Definition

The electric field, denoted as \mathbf{E} , is defined as the force per unit charge:

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

where:

- \mathbf{E} is the electric field (in newtons per coulomb, N/C),
- \mathbf{F} is the electric force experienced by a test charge,
- q is the charge experiencing the force.

This definition means that if we place a small positive charge (*test charge*) in an electric field, the force it experiences tells us the strength and direction of the field at that location.

5.3 Direction of the Electric Field

The direction of the electric field at a point is the direction of the force that would act on a small **positive** test charge placed at that point:

- If the field is created by a **positive** charge, the electric field points outward (away from the charge).
- If the field is created by a **negative** charge, the electric field points inward (toward the charge).

Fig. 8 shows a depiction of the electric field for a positive charge and a negative charge. The lines are called "field lines" and will be discussed later in this lecture.

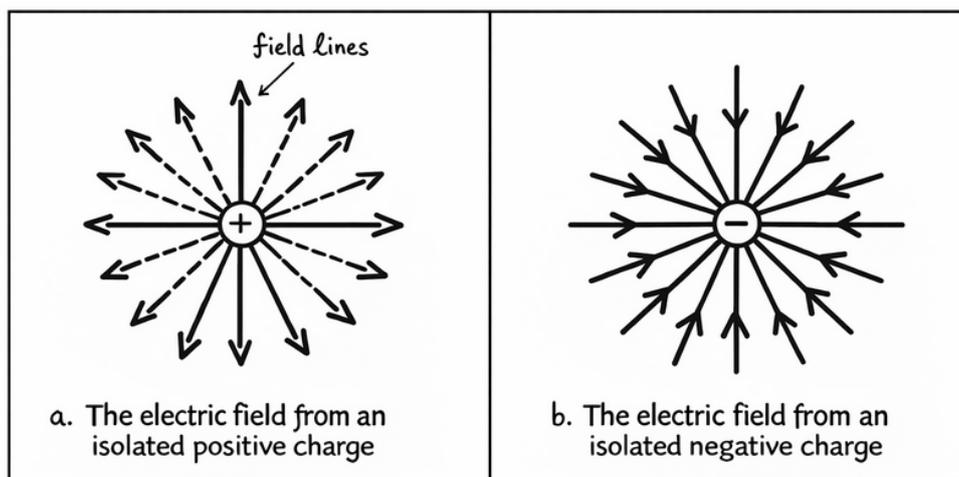


Figure 8: Electric field lines for a positive charge (left) and a negative charge (right). The field points outward for a positive charge and inward for a negative charge

5.4 Electric Field Due to a Point Charge

The electric field created by a single point charge is given by:

$$\mathbf{E} = k_e \frac{|q|}{r^2} \hat{r} \quad (3)$$

where:

- $k_e = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$ is Coulomb's constant,
- q is the charge creating the field,
- r is the distance from the charge to the point of interest.
- \hat{r} is the direction from the charge to the point of interest.

This equation shows that the electric field gets stronger as you get closer to the charge and weaker as you move farther away.

At a particular point in space, each of the surrounding charges contributes to the net electric field that exists there. To determine the net field, it is necessary to obtain the various contributions separately and then find the vector sum of them all. Such an approach is an illustration of the principle of linear superposition, as applied to electric fields. The following example demonstrates this.

5.5 Example: The Superposition of Electric Fields

Consider two charged objects, A and B, as shown in Fig. 9. Each contributes as follows to the net electric field at point P at the coordinates (0,0). $\mathbf{E}_A = 3.00\text{N}/\text{C}$ directed to the right, and $\mathbf{E}_B = 2.00\text{N}/\text{C}$ directed downward. Thus, \mathbf{E}_A and \mathbf{E}_B are perpendicular. What is the net electric field at P?

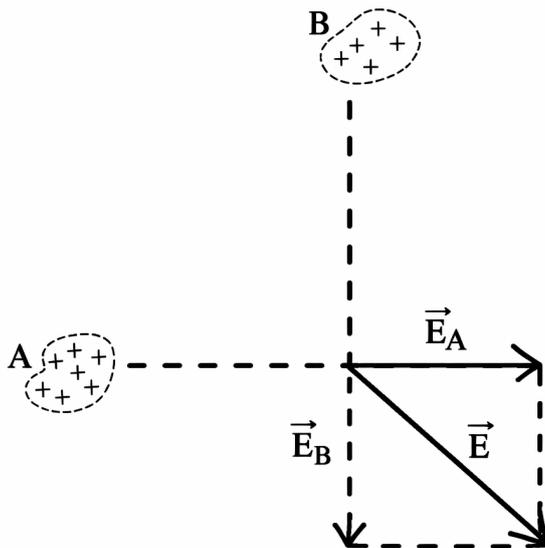


Figure 9: Two charged objects result in a net Electric Field.

Solution

The magnitude of the net electric field is

$$E = \sqrt{E_A^2 + E_B^2} = \sqrt{(3.00\text{N}/\text{C})^2 + (2.00\text{N}/\text{C})^2} = \boxed{3.61\text{N}/\text{C}}$$

The direction of \mathbf{E} is given by the angle $360^\circ - \theta$. Why 360° ? Because the net electric field points in quadrant IV, which is between 270° and 360° . We can find θ using the inverse tangent function:

$$\theta = \tan^{-1} \left(\frac{E_B}{E_A} \right) = \tan^{-1} \left(\frac{2.00 \text{ N/C}}{3.00 \text{ N/C}} \right) = 33.7^\circ$$

So, \mathbf{E} points at an angle of $360^\circ - 33.7^\circ = 326.3^\circ$ from the positive x -axis.

5.6 Example: The Electric Field of a Point Charge



Figure 10: (a) At location P , a positive test charge q_0 experiences a repulsive force \mathbf{F} due to the positive point charge q .

There is an isolated point charge of $q = +15 \mu\text{C}$ in a vacuum on the left in Fig. 10. Using a test charge of $q_0 = +0.80 \mu\text{C}$, determine the electric field at point P , which is 0.20 m away.

Following the definition of the electric field, we place the test charge q_0 at point P , determine the force acting on the test charge, and then divide the force by the test charge.

Solution

Coulomb's law gives the magnitude of the force:

$$F = k_e \frac{|q_0||q|}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.80 \times 10^{-6} \text{ C})(15 \times 10^{-6} \text{ C})}{(0.20 \text{ m})^2} = 2.7 \text{ N}$$

Eq. 3 gives the magnitude of the electric field:

$$E = \frac{F}{|q_0|} = \frac{2.7 \text{ N}}{0.80 \times 10^{-6} \text{ C}} = \boxed{3.4 \times 10^6 \text{ N/C}}$$

The electric field \mathbf{E} points in the *same direction* as the force \mathbf{F} on the positive test charge. Since the test charge experiences a force of repulsion directed to the right, the electric field vector also points to the right, as Fig. 10 shows.

Check yourself: If q were negative, draw a diagram similar to Fig. 10. What would change if q_0 at P were also negative?

5.7 The Parallel Plate Capacitor

A **capacitor** is a device used to store electric charge. The simplest type of capacitor is a **parallel plate capacitor**, which consists of two flat conducting plates placed near each other, separated by air or another insulating material as shown in Fig. 11.

When charge is placed on a parallel plate capacitor, one plate becomes positively charged, and the other becomes negatively charged by the same amount. This creates a separation of charge, which results in an electric field between the plates.

If we place a total charge Q on the plates, the positive plate has charge $+Q$ and the negative plate has charge $-Q$. The amount of charge a capacitor can store depends on:

- The **size of the plates** – larger plates can hold more charge.
- The **distance between the plates** – closer plates create a stronger attraction between opposite charges.
- The **material between the plates** – different materials can influence how much charge is stored.

Since opposite charges attract, an **electric field** forms in the space between the plates. The magnitude of this electric field is given by:

$$E = \frac{Q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0} \quad (4)$$

where:

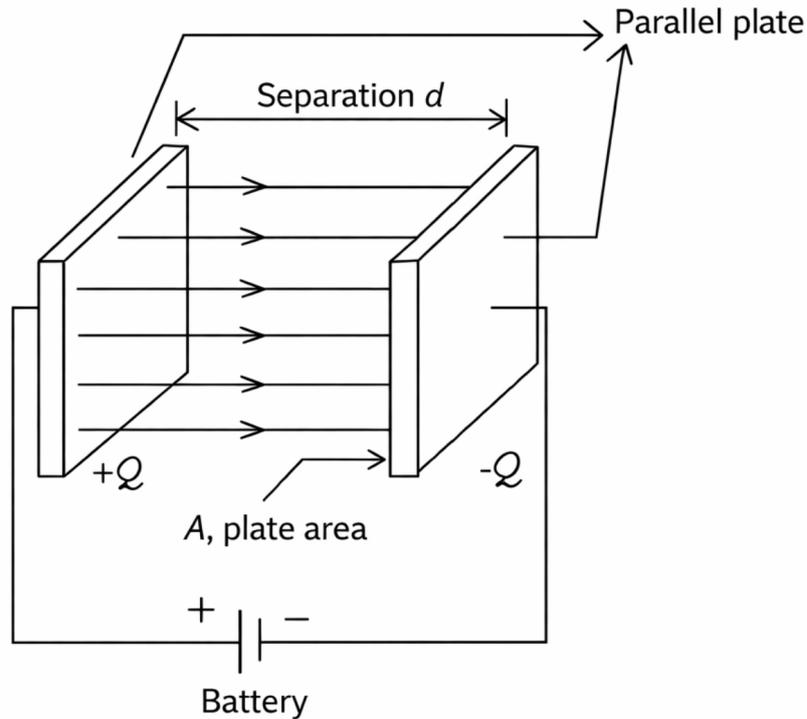


Figure 11: Parallel Plate Capacitor

- E is the magnitude of the electric field between the plates,
- Q is the charge on one of the plates,
- A is the area of each plate,
- $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ is the permittivity of free space,
- $\sigma = Q/A$ is the **surface charge density**, which represents charge per unit area on the plates.

This equation tells us that the electric field depends only on the charge per unit area and not on the separation distance (as long as the plates are close together and the field is uniform).

A good way to think about a capacitor is like a bucket holding charge. The larger the bucket (plate area), the more charge it can hold. If the plates are too far apart, it is harder to store charge, just as a deep bucket with a small opening would be harder to fill.

5.8 Electric Field Lines

Electric field lines are imaginary lines drawn to represent the direction of the electric field at different points in space. They follow these important rules:

- Electric field lines point **away from positive charges** and **toward negative charges**.
- The direction of the field at any point is *tangent* to the field line at that location.
- The density of field lines (how close they are together) indicates the **strength** of the electric field. More lines mean a stronger field.
- Field lines never cross, since the electric field at any point has only one direction.

Field Lines for Different Charge Configurations:

- **Single Positive or Negative Charge:** The field lines radiate outward from a positive charge and inward toward a negative charge. We have already seen this in Fig. 8.
- **Two Opposite Charges (Dipole):** The field lines begin on the positive charge and end on the negative charge, forming curved paths as seen in Fig. 12.

- **Two Like Charges:** The field lines repel each other, showing a region of low field strength between them as seen in Fig. 13.

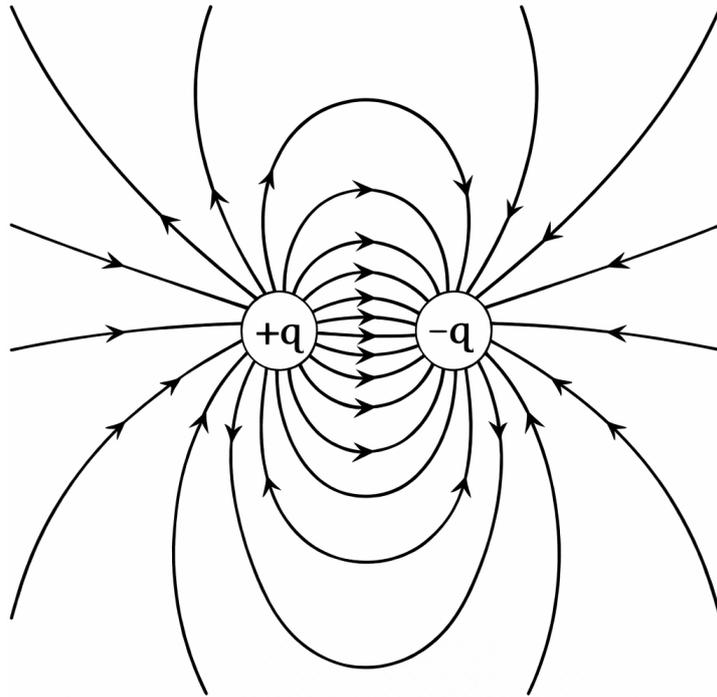


Figure 12: The electric field lines of an electric dipole are curved and extend from the positive to the negative charge. At any point, such as 1, 2, or 3, the field created by the dipole is tangent to the line through the point.

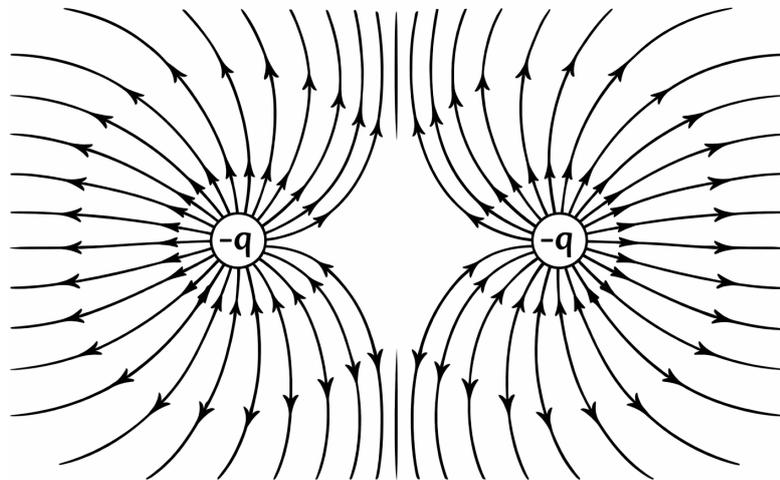


Figure 13: The electric field lines for two identical positive point charges. If the charges were both negative, the directions of the lines would be reversed.

5.9 The Electric Field Inside a Conductor

One of the fundamental properties of conductors is that the electric field inside a conductor in electrostatic equilibrium is zero. This leads to the important concept of **electrostatic shielding**, which is used in many real-world applications.

When a conductor is placed in an external electric field, free electrons inside the conductor move in response to the field. This movement continues until the charges redistribute themselves in such a way that they cancel out any electric field within the material of the conductor. As a result:

- Any **net charge** on a conductor resides on its **outer surface**.

- The **electric field inside the conductor is zero**.
- The conductor acts as a barrier, preventing external fields from influencing the space inside it.

This phenomenon is known as **electrostatic shielding** and is why sensitive electronic equipment is often enclosed in conductive shells.

A **Faraday cage** is a practical application of electrostatic shielding. It is a conducting enclosure that blocks external electric fields. Some examples include:

- **Car bodies:** If a lightning bolt strikes a car, the electric field inside remains zero, protecting passengers.
- **Microwave ovens:** The metal grid in the door prevents microwaves from escaping.
- **Shielded cables:** Coaxial cables use a conducting outer layer to prevent interference.

6 Gauss' Law

A charge distribution is a collection of charges that are spread out over a region. Gauss' law describes the relationship between a charge distribution and the electric field it produces.

To understand Gauss' Law, we first introduce the idea of **electric flux**, which measures how much an electric field passes through a surface. Flux depends on:

- The **strength** of the electric field E .
- The **area** of the surface through which the field passes.
- The **direction** of the field relative to the surface.

Consider a point charge. The electric field it produces is given by Eq. 3. We now place this point charge at the center of an imaginary spherical surface of radius r , as shown in Fig. 14. Such a surface is called a **Gaussian Surface**.

Notice in Fig. 14, the surface area patch ΔA . We can define the **electric flux** through this patch as $\Delta\Phi_E = E\Delta A \cos\theta$, where θ is the angle between the electric field vector and the normal vector to the surface at that patch. In this case, since the electric field is radial and the normal vector to the surface is also radial, $\theta = 0^\circ$ and $\cos\theta = 1$. Thus, $\Delta\Phi_E = E\Delta A$.

Imagine covering the sphere with many tiny patches of area ΔA_i . Each patch contributes a small amount of electric flux, and the total flux through the entire spherical surface is the sum of the flux through all the patches. Since the electric field has the same magnitude at every point on the surface of the sphere (because it depends only on r), we can factor E out of the sum:

$$\Phi_E = \sum_i E\Delta A_i = E \sum_i \Delta A_i = EA$$

The sum of all of the little patches $\sum_i \Delta A_i$ is just the total surface area of the sphere, which is $A = 4\pi r^2$! We already know from Eq. 3 that the electric field for a point charge is,

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

which gives us the electric field everywhere on the surface of the gaussian sphere. We know that the area of a sphere is just $A = 4\pi r^2$ so

$$E = \frac{q}{A\epsilon_0},$$

and now we can write **Gauss's law for a point charge** as,

$$\Phi_E = EA = \frac{q}{\epsilon_0} \tag{5}$$

This result indicates that, aside from the constant ϵ_0 , the electric flux Φ_E depends only on the charge q within the Gaussian surface and is independent of the radius r of the surface. Using calculus this result can be proven for any closed surface that surrounds the charge q . This leads to the general result known as Gauss's Law:

The electric flux through any closed surface is equal to the net charge inside the surface, Q_{inside} , divided by ϵ_0 .

$$\Phi_E = \frac{Q_{inside}}{\epsilon_0} \quad (6)$$

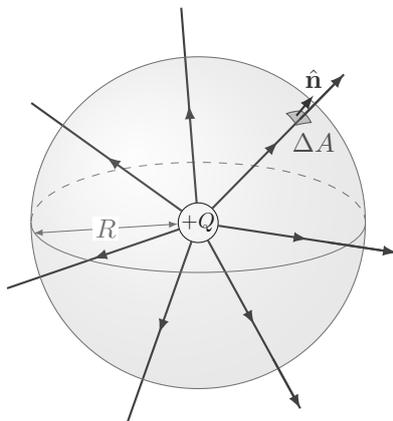


Figure 14: The electric flux through a small patch of area ΔA on the Gaussian surface. The electric field \mathbf{E} is tangent to the field line and points outward from the charge $+Q$. The normal vector $\hat{\mathbf{n}}$ is perpendicular to the surface and also points outward.

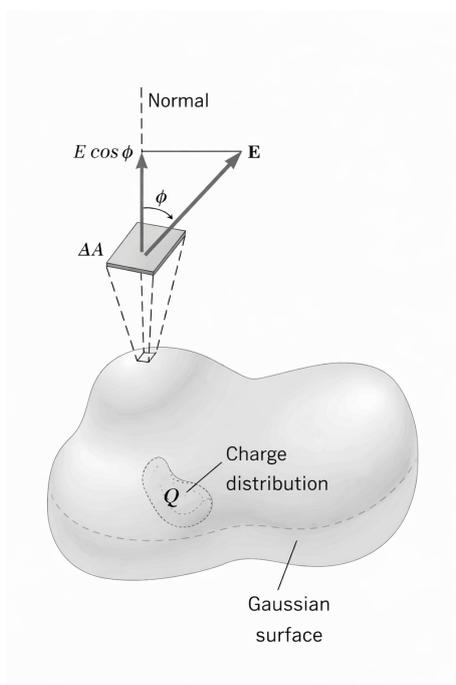


Figure 15: An irregularly shaped Gaussian surface. The electric flux through this surface is still given by Gauss's law, $\Phi_E = Q_{inside}/\epsilon_0$.

So even if the Gaussian surface is something irregular, like in Fig. 15, Eq. 6 still describes the electric flux through its surface. What does this mean? It means that we can generalize it to represent any distribution of charges within any arbitrary shaped Gaussian surface! In practice, the technique is useful only in a limited number of cases in which there is a high degree of symmetry, such as spheres, cylinders, or planes. If the Gaussian surface is chosen so that the electric field is constant everywhere on it, the electric field can be computed with,

$$\Phi_E = EA \quad (7)$$

Putting Eq. 7 into Eq. 6 gives us a powerful tool to compute the electric field for highly symmetric charge distributions:

$$EA = \frac{Q_{inside}}{\epsilon_0}. \quad (8)$$

6.1 Example: The Electric Field of a Charged Spherical Shell

A spherical conducting shell of inner radius a and outer radius b carries a total charge $+Q$ distributed on the surface of a conducting shell as shown in Fig. 16(I). The quantity Q is taken to be positive. (a). Find the electric field in the interior of the conducting shell for $r < a$ as shown in Fig. 16(II) and (b). the electric field outside the shell for $r > b$ as shown in Fig. 16(III). (c) If an additional charge of $-2Q$ is placed at the center, find the electric field for $r > b$. (d). What is the distribution of charge on the sphere in part (c)?

Solution:

a. Find the electric field for $r < a$.

$$EA = E(4\pi r^2) = \frac{Q_{inside}}{\epsilon_0} = 0 \rightarrow E = 0$$

b. Find the electric field for $r > b$.

$$EA = E(4\pi r^2) = \frac{Q_{inside}}{\epsilon_0} = \frac{Q}{\epsilon_0} \rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$

c. Now an additional charge of $-2Q$ is placed at the center of the sphere. Compute the new electric field outside the sphere, for $r > b$.

$$EA = E(4\pi r^2) = \frac{Q_{inside}}{\epsilon_0} = \frac{Q - 2Q}{\epsilon_0} \rightarrow E = -\frac{Q}{4\pi\epsilon_0 r^2}$$

d. Find the charge distribution on the sphere for part (c).

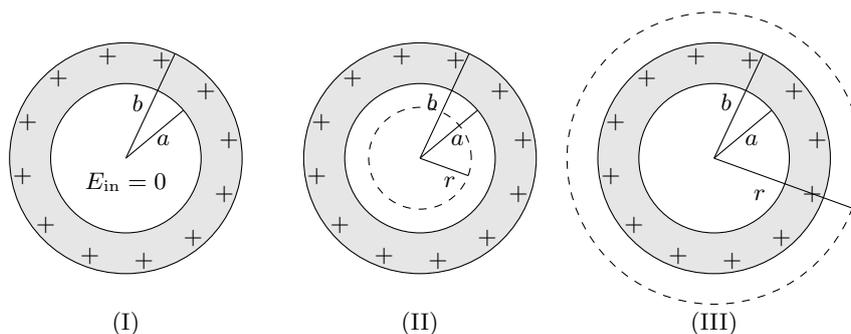


Figure 16: (a). A cross-section of a spherical conducting shell of inner radius a and outer radius b carries a total charge $+Q$ distributed on the surface of a conducting shell. The electric field is 0 inside the conducting shell. (b). A Gaussian surface for $r < a$. (c). A Gaussian surface for $r > b$.

$$EA = E(4\pi r^2) = \frac{Q_{inside}}{\epsilon_0} = \frac{Q_{center} + Q_{innersurface}}{\epsilon_0}$$

The charge on the inner surface of the shell:

$$Q_{center} + Q_{innersurface} = 0$$

because the electric field in the conductor is zero. So

$$Q_{innersurface} = -Q_{center} = 2Q$$

Find the charge on the outer surface, noting that the inner and outer surface charges must sum to $+Q$:

$$Q_{outersurface} + Q_{innersurface} = Q$$

$$Q_{outersurface} = -Q_{innersurface} + Q = -Q$$

6.2 Example: A Nonconducting Plane Sheet of Charge

Find the electric field above and below a nonconducting infinite plane sheet of charge with uniform positive charge per unit area σ as shown in Fig. 6.2a. **Solution:**

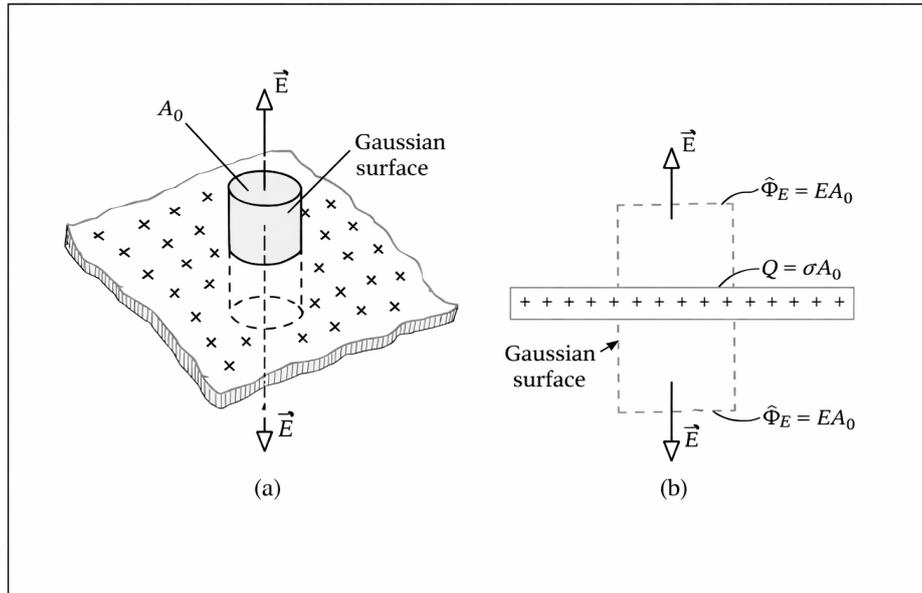


Figure 17: (a). A cylindrical Gaussian surface penetrating an infinite sheet of charge. (b). A cross-section of the same Gaussian cylinder. The flux through each end of the Gaussian surface is EA_0 . There is no flux through the sides of the cylindrical surface.

By symmetry the electric field must be perpendicular to the plane and directed away from it on either side as shown in Fig. 6.2b. For the Gaussian surface, choose a small cylinder with axis perpendicular to the plane, each end having area of A_0 . No electric field lines pass through the curved surface of the cylinder, only through the two ends, which have total area $2A_0$. Apply Gauss's law, using Fig. 6.2b.

(a). Find the electric field above and below a plane of uniform charge. Using Gauss's law:

$$EA = \frac{Q_{inside}}{\epsilon_0}$$

The total charge inside the Gaussian cylinder is the charge density times the cross-sectional area:

$$Q_{inside} = \sigma A_0$$

The electric flux comes entirely from the two ends, each having area A_0 . Substitute $A = 2A_0$ and Q_{inside} and solve for E.

$$E = \frac{\sigma A_0}{(2A_0)\epsilon_0} = \frac{\sigma}{2\epsilon_0}$$

This is the magnitude of the electric field. Find the z-component of the field above and below the plane. The electric field points away from the plane, so it's positive above the plane and negative below the plane.

$$E_z = \frac{\sigma}{2\epsilon_0}, z > 0$$

$$E_z = -\frac{\sigma}{2\epsilon_0}, z < 0$$

7 Electric Potential Energy and the Electric Potential

7.1 Electric Potential Energy

Just as gravity can do work on masses by moving them through a gravitational field, electric forces can do work on charges by moving them through an electric field. The concept of **potential energy** applies in both situations.

In mechanics, the gravitational potential energy of a mass m at a height h above some reference point is given by:

$$U_g = mgh$$

where:

- m is the mass (in kg),
- g is the gravitational acceleration (9.8 m/s²),
- h is the height above the reference level (in meters).

This expression tells us the energy an object has due to its position in a gravitational field.

Similarly, in electrostatics, the electric potential energy of a charge q at a location where the electric potential is V is:

$$U_e = qV$$

where:

- q is the electric charge (in coulombs),
- V is the electric potential (in volts).

This tells us how much energy the charge has due to its position in an electric field. As further analogy, just as lifting a mass increases its gravitational potential energy, moving a positive charge against the electric field increases its electric potential energy. Both types of potential energy represent stored energy due to position in a field.

Table 7.1 summarizes the analogies between gravitational and electric systems. Both types of potential energy represent stored energy due to position in a field, and they can be converted into kinetic energy when the object or charge moves under the influence of the respective forces.

Gravitational System	Electric System
Mass m	Charge q
Gravitational field g	Electric field E
Potential energy $U_g = mgh$	Potential energy $U_e = qV$
Potential gh	Potential V
Force $F = mg$	Force $F = qE$

Table 1: Analogies between gravitational and electric systems.

7.2 Electric Potential Difference and Work

When a positive test charge q_0 is placed in an electric field \mathbf{E} , it experiences an electric force $\mathbf{F} = q_0\mathbf{E}$. If the charge is allowed to move, this force does work on the charge, changing its electric potential energy. Consider Fig. 18. The work done by the electric field on the charge as it moves from point A to point B is equal to the negative of the change in electric potential energy:

$$W_{AB} = -\Delta U_e = -(U_{e,B} - U_{e,A}) = -(q_0V_B - q_0V_A) = -q_0(V_B - V_A) = -q_0\Delta V$$

The minus sign tells us that the work done by the electric field on the charge is equal to the decrease in the charge's electric potential energy. If the charge moves in the direction of the electric field, it loses potential energy and gains kinetic energy. If it moves against the electric field, it gains potential energy and loses kinetic energy.

Therefore, the **electric potential difference** (ΔV) between two points A and B is defined as the work done per unit charge by the electric force to move a charge from point A to point B :

$$\Delta V = V_B - V_A = -\frac{W_{AB}}{q_0}$$

Electric potential difference is measured in **volts (V)**: 1 V = 1 Joule/Coulomb. This means that if 1 joule of work is done to move 1 coulomb of charge between two points, the potential difference between them is 1 volt.

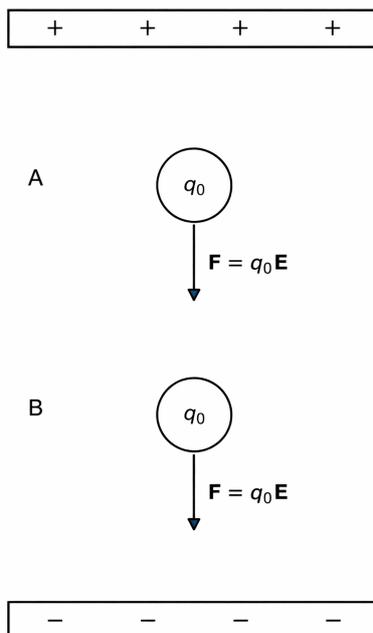


Figure 18: Because of the electric field \mathbf{E} , an electric force, $\mathbf{F} = q_0\mathbf{E}$, is exerted on a positive test charge $+q_0$. Work is done by the force as the charge moves from A to B .

Note: Many students get confused about the sign. I believe it is because they are confusing **work done by the electric field** with **work done by an external agent**. The work done by the electric field is negative of the change in potential energy, while the work done by an external agent to move the charge against the electric field is positive of the change in potential energy. In other words, if you push a charge against the electric field, you are doing positive work on it, and it gains potential energy. If you let it go and it moves with the electric field, the electric field does positive work on it, and it loses potential energy.

Example 1: Work, Electric Potential Energy, and Electric Potential

In Figure 18, the work done by the electric force as the test charge $q_0 = +2.0 \times 10^{-6}$ C moves from point A to point B is

$$W_{AB} = +5.0 \times 10^{-5} \text{ J.}$$

- (a) Find the difference in electric potential energy:

$$\Delta U_e = U_{e,B} - U_{e,A}$$

- (b) Determine the potential difference between the points:

$$\Delta V = V_B - V_A$$

Reasoning

The work done by the electric force when the charge moves from A to B is:

$$W_{AB} = U_{e,A} - U_{e,B}$$

which means the change in electric potential energy is:

$$\Delta U_e = U_{e,B} - U_{e,A} = -W_{AB}.$$

The potential difference is defined as:

$$\Delta V = V_B - V_A = \frac{\Delta U_e}{q_0} = \frac{U_{e,B} - U_{e,A}}{q_0}.$$

Solution

(a) The difference in electric potential energy is:

$$U_{e,B} - U_{e,A} = -W_{AB} = -5.0 \times 10^{-5} \text{ J} \quad (19.1)$$

The negative sign indicates that the charge has more electric potential energy at point A than at point B .

(b) The potential difference between A and B is:

$$V_B - V_A = \frac{U_{e,B} - U_{e,A}}{q_0} = \frac{-5.0 \times 10^{-5} \text{ J}}{2.0 \times 10^{-6} \text{ C}} = -25 \text{ V} \quad (19.4)$$

This result means that the electric potential is higher at point A than at point B .

Conceptual Example: The Car Battery

A classic example of electric potential and electric potential energy is shown in Figure 19, which depicts a 12-volt car battery powering a headlight.

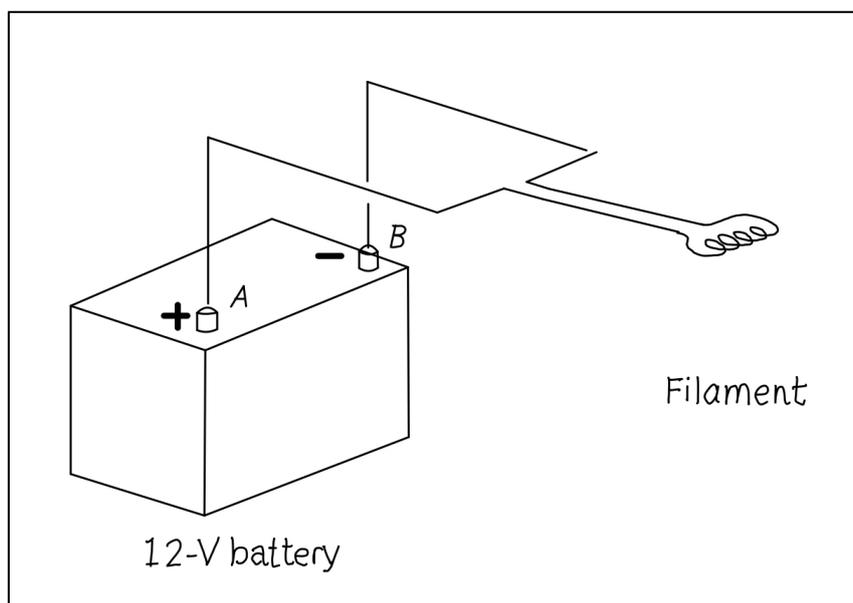


Figure 19: A headlight connected to a 12-V battery.

The positive terminal (point A) is 12 volts higher in electric potential than the negative terminal (point B); that is, $V_A - V_B = 12 \text{ V}$. Positive charges would be pushed away from the positive terminal and flow through the wire and headlight toward the negative terminal. As they pass through the headlight, their electric potential energy is converted almost entirely into heat and light, causing the filament to glow. When the charges reach the negative terminal, they have little or no potential energy left. The battery then does work to move them back to the positive terminal, using its internal chemical energy to restore their electric potential energy and repeat the cycle.

7.3 The Electric Potential Difference Created by Point Charges

Electric potential is a scalar quantity that describes the amount of electric potential energy per unit charge at a point in space. For a point charge, the electric potential V at a distance r from the charge q is given by:

$$V = \frac{kq}{r}$$

where $k_e = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ is Coulomb's constant.

In Figure 20, a positive test charge $+q_0$ is placed near another positive charge $+q$. Because like charges repel, the test charge experiences a force \mathbf{F} pushing it away from $+q$. If the test charge moves from point A to point B , it moves to a region of lower electric potential – similar to a mass sliding downhill under gravity.

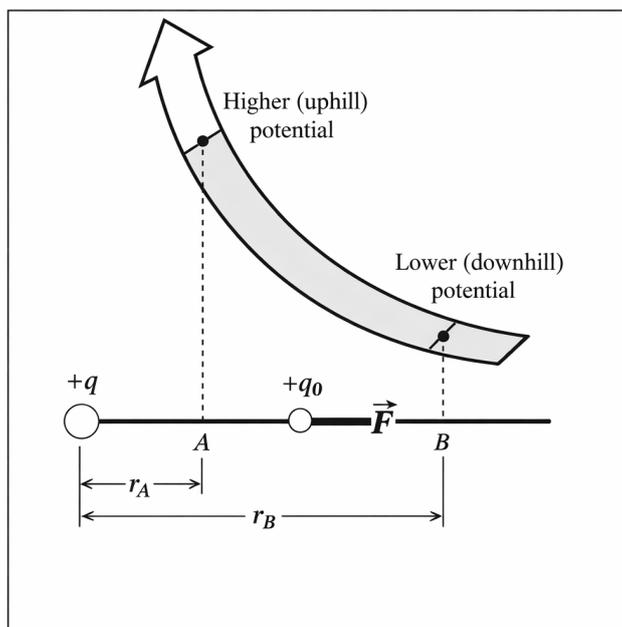


Figure 20: A positive test charge $+q_0$ experiences a repulsive force from another positive charge $+q$, moving from point A to point B under the influence of the electric field.

The electric potential difference between two points A and B in the field of a point charge is:

$$\Delta V = V_B - V_A = \frac{kq}{r_B} - \frac{kq}{r_A} = -\frac{W_{AB}}{q_0} \quad (9)$$

Since $r_B > r_A$, this difference is negative, indicating a drop in potential as the test charge moves away from the source charge.

Positive charges naturally move from regions of high potential to low potential – just like a ball rolling downhill. In this case, the electric potential is highest closer to the source charge and decreases with distance.

Note: The symbol V in the equation $V = kq/r$ does not represent an absolute value of electric potential. Instead, it tells us how much higher the electric potential is at a distance r from a point charge compared to a point very far away (at infinity). We are using the standard assumption that the electric potential at infinity is zero, so V really represents a **potential difference** between the point at distance r and infinity. If we want to find the potential difference between two points, A and B , we subtract the two values as we did in Eq. 9. Even though both V_A and V_B are defined relative to infinity, their difference ΔV is completely meaningful – it tells us how much more (or less) electric potential energy per unit charge exists at one point compared to the other. Since both values use the same reference (infinity), the reference cancels out when we compute the difference.

Example: The Potential Energy of a Group of Charges

Three point charges are initially located infinitely far apart. They are then brought together and placed at the corners of an equilateral triangle, as shown in Figure 7.3. Each side of the triangle has a length of 0.50 m. Determine the total electric potential energy U_e of the system once all charges are assembled.

Reasoning: We calculate the electric potential energy by assembling the charges one at a time, finding the potential at each step due to the previously placed charges. The total electric potential energy is the sum of the energies required to bring in each charge.

Step 1: Place q_1 No work is needed to bring in the first charge from infinity, since there are no other charges present:

$$U_{e,1} = 0 \text{ J}$$

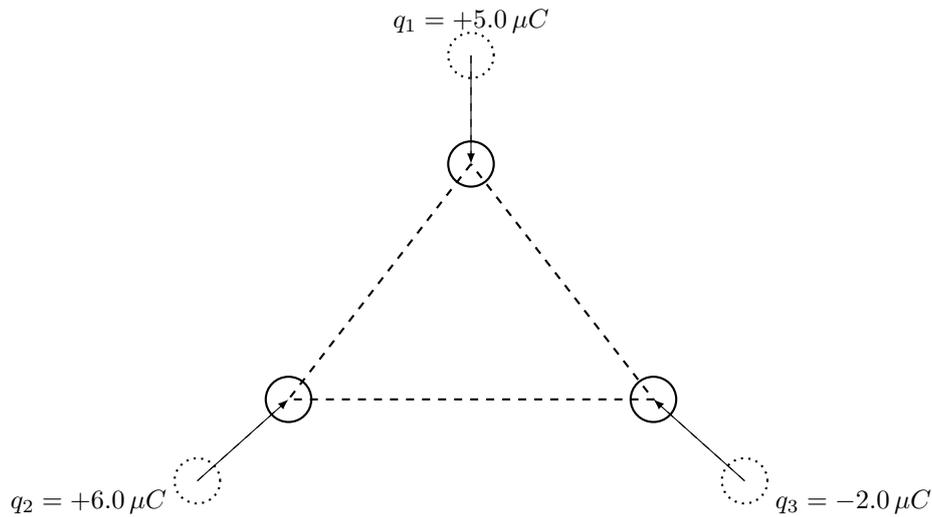


Figure 21: Three point charges are placed on the corners of an equilateral triangle.

Step 2: Place q_2 The potential at the location of q_2 due to q_1 is:

$$V_1 = \frac{kq_1}{r} = \frac{(8.99 \times 10^9)(5.0 \times 10^{-6})}{0.50} = 9.0 \times 10^4 \text{ V}$$

Then the potential energy of q_2 is:

$$U_{e,2} = q_2 V_1 = (6.0 \times 10^{-6})(9.0 \times 10^4) = 0.54 \text{ J}$$

Step 3: Place q_3 The potential at the location of q_3 is the sum of the potentials due to q_1 and q_2 :

$$V_1 + V_2 = \frac{kq_1}{r} + \frac{kq_2}{r} = \frac{(8.99 \times 10^9)(5.0 \times 10^{-6})}{0.50} + \frac{(8.99 \times 10^9)(6.0 \times 10^{-6})}{0.50} = 2.0 \times 10^5 \text{ V}$$

Then the potential energy of q_3 is:

$$U_{e,3} = q_3(V_1 + V_2) = (-2.0 \times 10^{-6})(2.0 \times 10^5) = -0.40 \text{ J}$$

Total Potential Energy of the System:

$$U_e = U_{e,1} + U_{e,2} + U_{e,3} = 0 + 0.54 - 0.40 = \boxed{+0.14 \text{ J}}$$

This total energy represents the work required to assemble the three charges from infinitely separated positions into the given triangular configuration.

7.4 Equipotential Surfaces

An equipotential surface is just what its name declares: a surface - real or imaginary - on which all points have the same electric potential. Consider any point on a sphere around a point charge as shown in Fig.7.4. All points have the same distance, r , so they all have the same potential $V = kq/r$. So how much work is needed to move a test charge, q_o along the surface of the sphere? Zero.

Consider two of the equipotential spheres around a positive point charge as shown in Fig. 7.4. At each location on either of the equipotential spheres the electric field is perpendicular to the surface and points outward in the direction of decreasing potential. How do we know that the second equipotential surface is at a lower potential? Well $r_2 > r_1$ and the electric potential is inversely related to the distance so a bigger r results in a lower potential for the same charge.

This perpendicular relation is valid whether or not the equipotential surfaces result from a positive or negative charge or have a spherical shape. **The electric field created by any charge or group of charges is everywhere perpendicular to the associated equipotential surfaces and points in the direction of decreasing potential.**

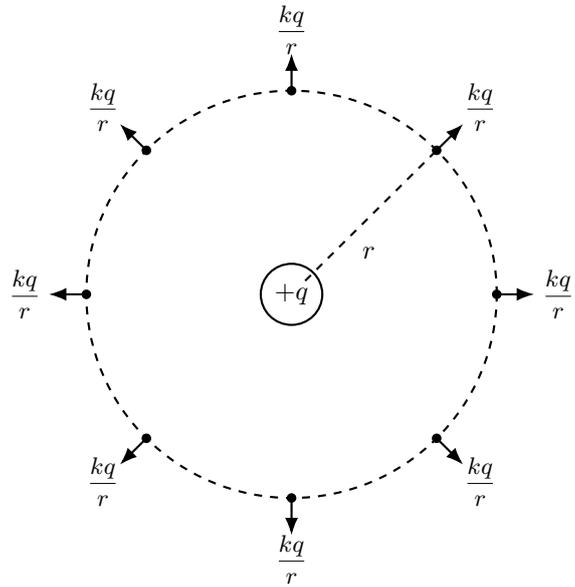


Figure 22: A cross-sectional diagram of a sphere with a point charge inside showing that at a distance r from the charge the electric potential doesn't change.

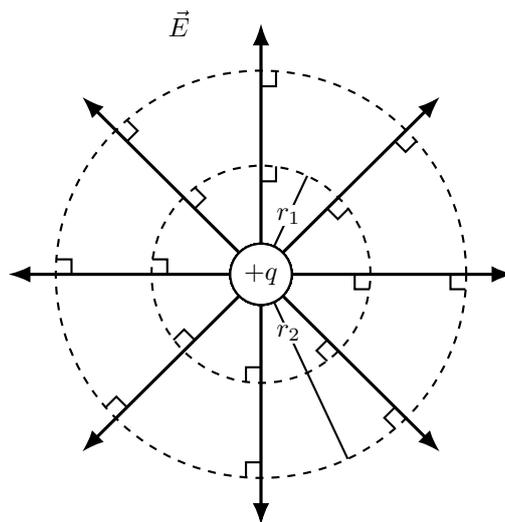


Figure 23: Two equipotential surfaces around a point charge. The first is a distance r_1 from the point charge and the second, r_2 . The arrows are electric field lines and are perpendicular to both equipotential surfaces.

Example: Work and Motion on Equipotential Surfaces

A positive test charge $q_0 = +1.0 \mu\text{C}$ is placed near a positive point charge $Q = +5.0 \mu\text{C}$. The test charge is moved from point A to point B , where:

- Point A lies on an equipotential surface where the potential is $V_A = 1200 \text{ V}$,
 - Point B lies on an equipotential surface where the potential is $V_B = 800 \text{ V}$.
- (a) How much work is done by the electric field as the test charge moves from point A to point B ?
- (b) Would the test charge move spontaneously from A to B ? Explain.
- (c) What would the work be if the charge moved along a path that stayed entirely on the equipotential surface at $V = 1200 \text{ V}$?

Solution

- (a) Use the relationship between potential difference and work:

$$W = -q_0(V_B - V_A)$$

Substituting values:

$$W = -(1.0 \times 10^{-6} \text{ C})(800 \text{ V} - 1200 \text{ V}) = -(1.0 \times 10^{-6})(-400) = +4.0 \times 10^{-4} \text{ J}$$

So, the electric field does $\boxed{+0.40 \text{ mJ}}$ of work on the charge.

- (b) Yes, the charge would move spontaneously from A to B because it's moving from a region of higher potential to lower potential. For a positive charge, the electric field pushes it in the direction of decreasing potential, doing positive work.
- (c) If the charge moves along an equipotential surface (e.g., around the charge but staying at $V = 1200 \text{ V}$), the potential difference is zero:

$$\Delta V = 0 \quad \Rightarrow \quad W = -q_0 \cdot 0 = 0$$

So the electric field does $\boxed{0 \text{ J}}$ of work. This confirms that no work is required to move along an equipotential surface.

7.5 Capacitors and Dielectrics

What Is Capacitance?

Capacitance is a measure of how much electric charge a capacitor can store for a given potential difference (voltage) across its plates. The basic relationship is:

$$C = \frac{Q}{\Delta V} \tag{10}$$

where:

- C is the capacitance (in farads, F),
- Q is the amount of charge stored (in coulombs, C),
- ΔV is the potential difference across the plates (in volts, V).

Simple Interpretation: Capacitance tells us how “charge-hungry” a capacitor is. A larger capacitance means the capacitor can store more charge at the same voltage.

Analogy: Water Tank

You can think of a capacitor like a water tank:

- Voltage ΔV is like water pressure.
- Charge Q is like the amount of water stored.

- Capacitance C is like the size of the tank.

Just like a larger tank stores more water at the same pressure, a capacitor with more capacitance stores more charge at the same voltage.

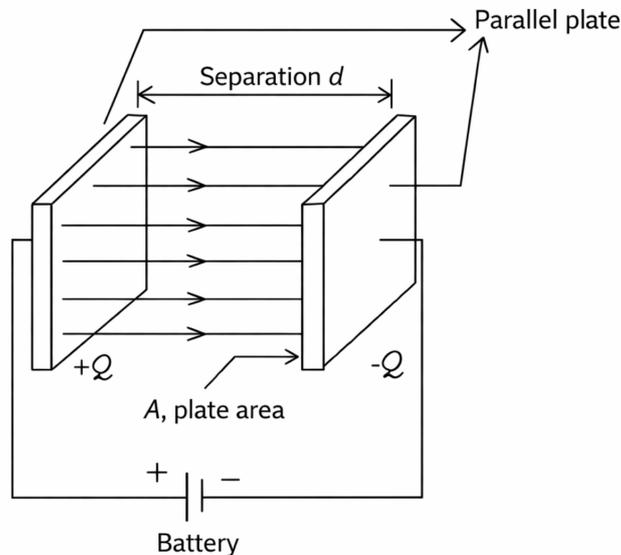


Figure 24: A basic parallel plate capacitor with charge $+Q$ and $-Q$ on each plate and a voltage ΔV across them.

7.5.1 Capacitance of a Parallel Plate Capacitor

The most basic capacitor is a **parallel plate capacitor**, which consists of two conducting plates separated by a small distance as shown in Fig. 24.

Recall how the change in electric potential relates to work,

$$\Delta V = \frac{W}{q}$$

which says that electric potential difference is the work done per unit charge to move a test charge between two points in an electric field.

Wait a minute – where did the negative sign go?

Earlier, the negative sign appeared because we calculated how potential changes when a charge moves in the direction of the electric field. In that situation, potential decreases, so ΔV was negative because we explicitly defined it as $V_{\text{final}} - V_{\text{initial}}$.

For a capacitor, however, we are not describing motion. We are simply comparing the potentials of the two plates. Since potential decreases in the direction of the electric field, the plate from which the field originates must be at a higher potential.

Therefore,

$$V_{\text{positive}} - V_{\text{negative}} = \frac{W_{\text{ext}}}{q}.$$

If we insisted on keeping the earlier directional definition of ΔV , we could instead write

$$|\Delta V| = \frac{W_{\text{ext}}}{q}.$$

Capacitance relates the amount of charge stored to how much potential difference exists between the plates. This is a physical property of the device, like the size of a bucket. That property cannot be negative.

Also, recall how F relates to E ,

$$F = qE$$

We know that,

$$W = F \cdot d = qE \cdot d$$

so,

$$\Delta V = \frac{W}{q} = \frac{qEd}{q} = Ed$$

or when rearranged,

$$E = \frac{\Delta V}{d}$$

Recall the electric field inside an empty parallel plate capacitor discussed earlier in Eq. 4,

$$E = \frac{q}{\epsilon_0 A}$$

Setting the last two equations equal,

$$\frac{\Delta V}{d} = \frac{q}{\epsilon_0 A}$$

Using $C = Q/\Delta v$, solving for ΔV , and plugging it into the above equation, we find that the capacitance depends on the geometry of the plates:

$$C = \epsilon_0 \frac{A}{d}$$

where:

- A is the area of one plate,
- d is the separation between the plates,
- $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$ is the vacuum permittivity.

Dielectrics and the Dielectric Constant

A **dielectric** is an insulating material placed between the plates of a capacitor. It increases the capacitance by reducing the electric field and preventing charge flow.

Consider the electric field on the interior of a parallel plate capacitor $E = q/\epsilon_0 A$. Let $\sigma = q/A$ and write the field as,

$$E_0 = \frac{\sigma}{\epsilon_0}$$

Now we insert a dielectric material. The dielectric molecules polarize, producing bound surface charges:

- Negative bound charge near the positive plate
- Positive bound charge near the negative plate

These bound charges create their own electric field, which opposes the original field as shown in Fig. 7.5.1 resulting in,

$$E = \frac{E_0}{\kappa}$$

The effect of dielectric is described by the **dielectric constant** κ , a unitless number that also affects capacitance,

$$C = \kappa \epsilon_0 \frac{A}{d}$$

where $\kappa > 1$ for all dielectrics. Some common dielectric constants can be found in Table 2.

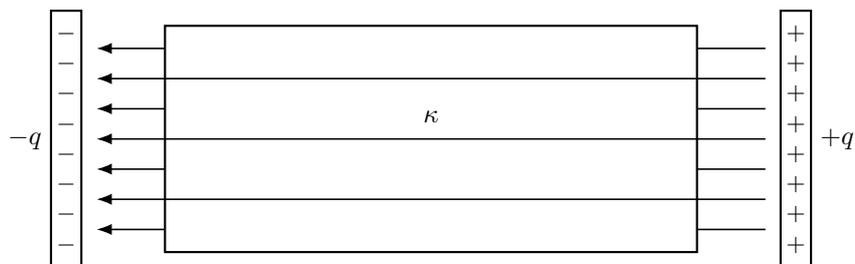


Figure 25: The surface charges on the dielectric reduce the electric field inside the dielectric.

Note: The dielectric does not “block” the electric field inside it. The field is still present, but the molecules in the material become polarized, creating induced charges that partially oppose the original field. In that sense, some of the field is “tied up” in aligning the material rather than contributing to the net field between the plates. The practical consequence is powerful: inserting a dielectric increases the capacitance, allowing the capacitor to store more charge (and energy) at the same applied voltage – which is why real capacitors are filled with dielectric materials rather than left as empty space.

Substance	Dielectric Constant, κ
Vacuum	1
Air	1.000 54
Teflon	2.1
Benzene	2.28
Paper (royal gray)	3.3
Ruby mica	5.4
Neoprene rubber	6.7
Methyl alcohol	33.6
Water	80.4

Table 2: Dielectric constants κ for various substances.

7.6 Energy Storage in a Capacitor

When a capacitor stores charge, it also stores energy. This energy comes from the work done by a battery (or other voltage source) to move charge from one plate to the other against the electric field.

As the capacitor charges, the voltage between the plates increases. That means each new bit of charge takes slightly more work to move than the previous one. The total work done in moving the full amount of charge Q is equal to the energy stored in the capacitor.

As a capacitor is charged from 0 to Q , the voltage across its plates increases linearly from 0 to V , because $V = \frac{Q}{C}$. The average voltage during the charging process is:

$$\bar{V} = \frac{0 + V}{2} = \frac{1}{2}V$$

From this point forward, V will denote the potential difference between the plates of the capacitor. Since there are only two relevant conductors, writing ΔV is unnecessary – the voltage across the capacitor is understood.

The total work done by the battery (which becomes stored energy in the capacitor) is:

$$U = Q \cdot \bar{V} = Q \cdot \left(\frac{1}{2}V\right) = \frac{1}{2}QV$$

This expression gives the energy stored in the capacitor.

Using the relationship $Q = CV$, we can substitute to get two additional forms:

$$U = \frac{1}{2}CV^2$$

$$U = \frac{1}{2} \frac{Q^2}{C}$$

These three forms are equivalent and useful depending on whether you know the charge, voltage, or capacitance.

Energy Stored in the Electric Field

We can also view this energy as being stored in the electric field between the plates. For a parallel plate capacitor, recall:

$$V = Ed \quad \text{and} \quad C = \kappa\epsilon_0 \frac{A}{d}$$

Substitute these into $U = \frac{1}{2}CV^2$:

$$U = \frac{1}{2} \left(\kappa\epsilon_0 \frac{A}{d} \right) (Ed)^2 = \frac{1}{2} \kappa\epsilon_0 AdE^2$$

Since Ad is the volume between the plates, the energy density (energy per unit volume) is:

$$\text{Energy density} = \frac{U}{\text{Volume}} = \frac{1}{2} \kappa\epsilon_0 E^2$$

This tells us how much energy is stored in a region of space where there is an electric field, even outside of capacitors.

Looking Ahead

Up to this point, we have developed the ideas of electric force, electric field, electric potential, and capacitance from first principles. We have treated charges as stationary and analyzed how energy is stored in electric fields.

Next, we will allow charges to move continuously and see how these same concepts – potential difference, energy, and field – govern the behavior of electric circuits. What we have called potential difference between two plates will become voltage across circuit elements, and the energy stored in fields will begin to flow through wires as electrical power.

Summary of Key Electrostatics Equations and Constants

Concept	Equation
Coulomb's Law (force between point charges)	$F = k_e \frac{ q_1 q_2 }{r^2}$
Electric Field from a Point Charge	$E = k_e \frac{ q }{r^2}$
Electric Force on a Charge in a Field	$\mathbf{F} = q\mathbf{E}$
Electric Potential from a Point Charge	$V = k_e \frac{q}{r}$
Potential Energy of a Charge in a Field	$U_e = qV$
Electric Potential Difference	$\Delta V = \frac{W}{q}$
Electric Field and Potential (uniform field)	$E = \frac{\Delta V}{d}$
Capacitance Definition	$C = \frac{Q}{\Delta V}$
Capacitance of a Parallel Plate Capacitor	$C = \kappa \epsilon_0 \frac{A}{d}$
Energy Stored in a Capacitor	$U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$
Energy Density in an Electric Field	$u = \frac{1}{2} \kappa \epsilon_0 E^2$
Gauss's Law (conceptual form)	$E \cdot A = \frac{q_{in}}{\epsilon_0}$

Table 3: Summary of key equations in electrostatics.

Constant	Symbol	Value
Coulomb's constant	k	$8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
Permittivity of free space	ϵ_0	$8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$
Elementary charge	e	$1.60 \times 10^{-19} \text{ C}$
Electron mass	m_e	$9.11 \times 10^{-31} \text{ kg}$
Proton mass	m_p	$1.67 \times 10^{-27} \text{ kg}$
Avogadro's number	N_A	$6.022 \times 10^{23} \text{ mol}^{-1}$
Speed of light (for context)	c	$3.00 \times 10^8 \text{ m/s}$

Table 4: Common constants used in electrostatics problems.