Electrostatics
Electric Forces, Fields, and Potentials

Chapter 18-19

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Physics 203

Electric Charge

What happens when you rub a balloon on your hair?

Electric Charge

The balloon has gained an invisible property that allows it to pull on other objects. But what is it? We can give it a name: *electric charge*. By understanding it, we might answer:

- Why do some objects attract while others repel?
- What does it mean for something to be “neutral”?
- How does charge move from one object to another?

Electrostatics is the study of electric charge at rest and the forces they exert on one another.

Electric Charge

Electric charge is a fundamental unit of matter that determines how objects interact through electrostatic forces.

Point Charge (q)

Charge of an individual particle (microscopic)

Quantized in units of the elementary charge

$$q = ne$$

$e = 1.602 \times 10^{-19} C$

System-Level Charge (Q)

Total charge of an object or system (macroscopic)

Sum of all individual charges

$$Q = \sum_i q_i$$

Conductors and Insulators

Materials can be classified into two broad categories:

Conductors

Materials that allow electric charge to move freely.

Examples: Metals, Saltwater, Plasma

At electrostatic equilibrium, excess charge lies on the surface, and the electric field inside the conductor is zero.

Insulators

Materials that do not allow charge to move freely.

Examples: Rubber, Plastic, Glass, Ceramics, Pure (distilled) water.

Excess charge stays localized, and electric fields can exist within the material.

Charging by Conduction

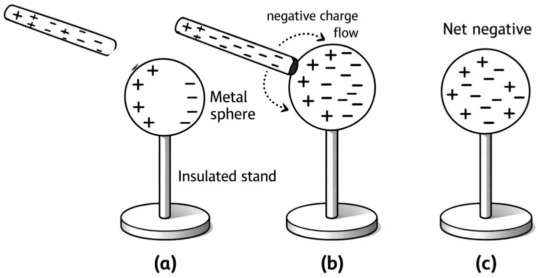


Figure 1: (a) A negatively charged rod is brought into contact with a neutral metal sphere mounted on an insulated stand. The sphere initially contains equal amounts of positive and negative charge. (b) When the rod touches the sphere, negative charge (electrons) flows between the rod and the sphere until they reach electrostatic equilbrium. (c) After the rod is removed, the sphere is left with an excess of negative charge.

Charging by Induction

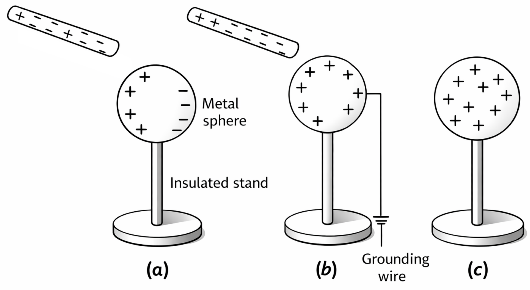


Figure 2: Charging by Induction Process: (a) A negatively charged rod is brought near a neutral metal sphere, causing charge separation. (b) The sphere is connected to the ground, allowing negative charge to flow away. (c) The ground connection is removed while the rod is still present. (d) The sphere is left with an induced positive charge after the rod is taken away.

Conservation of Charge

One of the fundamental principles of physics is the **law of charge conservation**:

> The total electric charge of an isolated system remains constant.
>
> Charge may be transferred, but it cannot be created or destroyed.

$$Q_{\text{final}} = Q_{\text{initial}}$$

This law applies to all physical processes, including:

- Conduction and induction
- Chemical reactions (e.g., batteries)
- Electric circuits

Consequence: In conductors, charge redistributes itself while preserving the total charge.

Coulomb's Law

The electrostatic force F between two point charges q_1 and q_2 separated by a distance r is given by:

$$F = k_e \frac{|q_1 q_2|}{r^2} \tag{1}$$

where:

F is the magnitude of the electrostatic force (in newtons, N),

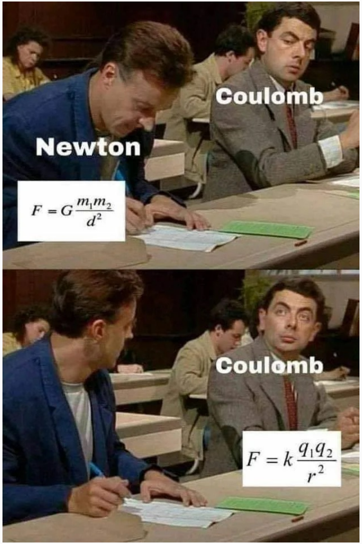
$|q_1|$ and $|q_2|$ are the magnitudes of the two charges (in coulombs, C),

r is the distance between the charges (in meters, m),

$k_e = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$ is Coulomb's constant.

If q_1 and q_2 have the same sign (both positive or both negative), the force is **repulsive**. If q_1 and q_2 have opposite signs, the force is **attractive**. The force acts along the straight line joining the two charges. Sometimes we will use $k_e = \frac{1}{4\pi\varepsilon_0}$ where ε_0 is the permittivity of free space, but the form of the equation remains the same. **Calculate ε_0!**

Wait a second...



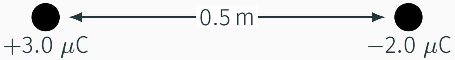
The form of Coulomb's law should look familiar to you,

$$F = k_e\frac{|q_1q_2|}{r^2}$$
$$F = G\frac{m_1m_2}{r^2}$$

Both are inverse square laws!

Example 1: 2 Point Charges in a Vacuum

Two point charges are placed in vacuum. Charge $q_1 = +3.0\ \mu$C is positioned at $x = 0$, and charge $q_2 = -2.0\mu$ C is placed at $x = 0.5$ m. Determine the magnitude and direction of the electrostatic force exerted by q_1 on q_2.



Solution

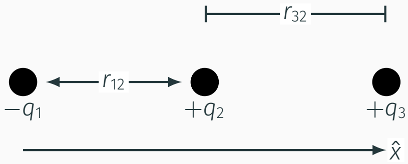
$$F = k_e \frac{|q_1 q_2|}{r^2} = (8.99 \times 10^9\ \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2) \frac{(3.0 \times 10^{-6}\ \mathrm{C})(2.0 \times 10^{-6}\ \mathrm{C})}{(0.5\mathrm{m})^2}$$
$$= 0.216\ \mathrm{N}$$

$q_1 > 0$ and $q_2 < 0 \rightarrow$ Force between them is attractive.

q_1 pulls q_2 to the left, q_2 pulls q_1 to the right

Example 2: 3 Charges in a Line

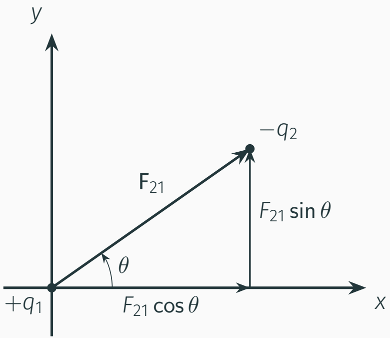
Consider three point charges arranged along a straight line.

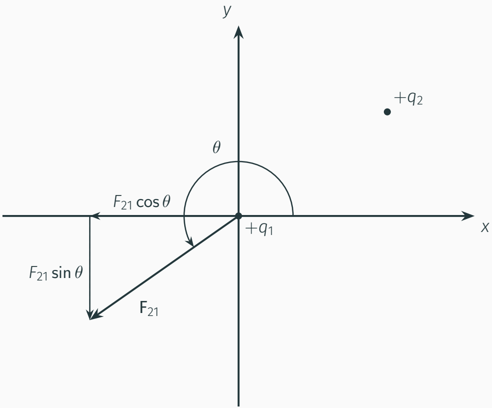


The net force on q_2 is the sum of forces due to q_1 and q_3. Each force is calculated using Coulomb's Law and then summed as vectors. For example,

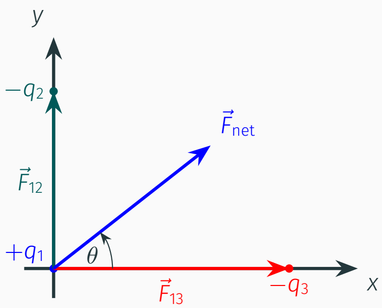
$$\vec{F}_{\text{net},2} = \vec{F}_{12} + \vec{F}_{32} = -k_e \frac{|q_1 q_2|}{r_{12}^2}\hat{x} - k_e \frac{|q_3 q_2|}{r_{32}^2}\hat{x} = -k_e \left(\frac{|q_1 q_2|}{r_{12}^2} + \frac{|q_3 q_2|}{r_{32}^2} \right) \hat{x}$$

Vector Components in 2D





Adding Vectors in 2D



Just like any other force, Coulomb forces are vectors! Using Coulomb's Law, each force is:

$$\vec{F}_{12} = k_e \frac{|q_1 q_2|}{r_{12}^2}\hat{y}, \quad \vec{F}_{13} = k_e \frac{|q_1 q_3|}{r_{13}^2}\hat{x}$$

Notice that both q_2 and q_3 are negative, q_1 is positive. Then we know,

$$\vec{F}_{net} = k_e \frac{|q_1 q_3|}{r_{13}^2}\hat{x} + k_e \frac{|q_1 q_2|}{r_{12}^2}\hat{y}$$

Written another way, its magnitude,

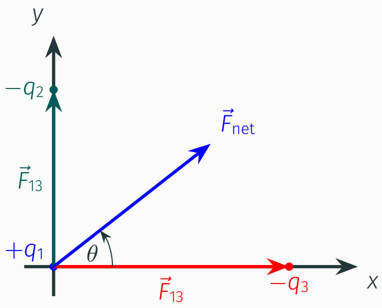
$$F_{net} = \sqrt{F_{net,x}^2 + F_{net,y}^2} = \sqrt{F_{13}^2 + F_{12}^2}$$

At an angle,

$$\theta = tan^{-1}\left(\frac{F_{net,y}}{F_{net,x}}\right) = tan^{-1}\left(\frac{F_{12}}{F_{13}}\right)$$

Example: Adding Vectors in 2D

If $q_1 = 3\mu C$, $q_2 = q_3 = -3\mu C$ and $r_2 = r_3 = 0.5\ m$, what is F_{net} and θ?



$$\vec{F}_{12} = k_e \frac{|q_1 q_2|}{r_{12}^2}\hat{y}, \quad \vec{F}_{13} = k_e \frac{|q_1 q_3|}{r_{13}^2}\hat{x}$$

Notice that both q_2 and q_3 are negative, q_1 is positive. Then we know,

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$$= \sqrt{F_{13}^2 + F_{12}^2}$$

At an angle,

$$\theta = tan^{-1}\left(\frac{F_{net,y}}{F_{net,x}}\right)$$
$$= tan^{-1}\left(\frac{F_{12}}{F_{13}}\right)$$

What is a Field?

A field is a way to describe how a force exists in space, even before anything experiences it.

Think of it as an invisible influence that fills a region and can exert a force on objects placed within it.

A field allows us to think about how one object creates a “presence” that extends outward, affecting other objects that come near.

Gravitational Field: Imagine some mass M, think of a point mass m like a probe, the gravitational field tells us what the gravitational force would be wherever in space we put m near M.

Electric Field: Likewise, imagine we have some charge Q, a test charge q like a probe, the Electric Field tells us what the electric force will be wherever in space we put q near Q.

The Electric Field in terms of Force

The electric field, denoted as $\vec{E}$, is defined as the force per unit charge:

$$\vec{E} = \frac{\vec{F}}{q}$$

where:

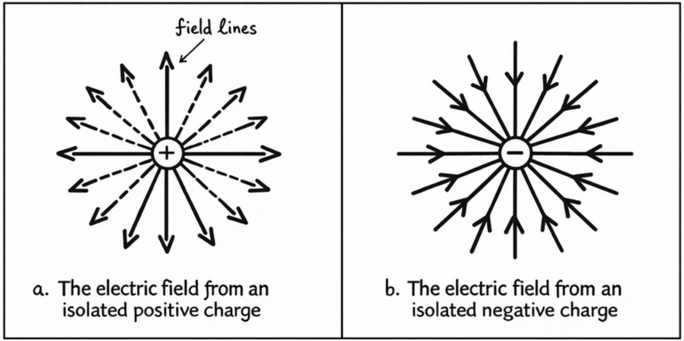
- $\vec{E}$ is the electric field (in newtons per coulomb, N/C),
- $\vec{F}$ is the electric force experienced by a test charge,
- q is the charge experiencing the force.

This definition means that if we place a small positive charge (*test charge*) in an electric field, the force it experiences tells us the strength and direction of the field at that location.

The Direction of an Electric Field

If the field is created by a **positive** charge, the electric field points outward (away from the charge).

If the field is created by a **negative** charge, the electric field points inward (toward the charge).



The Electric Field on a Point Charge

The magnitude of the electric field created by a single point charge is given by:

$$E = k_e \frac{|Q|}{r^2}$$

where:

- $k_e = 8.99 \times 10^9$ N·m^2/C^2 is Coulomb's constant,
- Q is the charge creating the field,
- r is the distance from the charge to the point of interest.

This equation shows that the electric field gets stronger as you get closer to the charge and weaker as you move farther away.

Example 5: The Electric Field of a Point Charge

There is an isolated point charge of $q = +15\,\mu\text{C}$ in a vacuum. Using a test charge of $q_0 = +0.80\,\mu\text{C}$, determine the electric field at point P, which is 0.20 m away to the right at point P.

Solution:

The electric field anywhere due to $+q$ is,

$$\vec{E} = k_e \frac{|q|}{r}\hat{r}$$

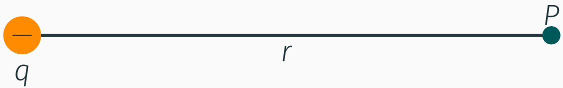
At point P, $\hat{r} = \hat{x}$ so,

$$\vec{E} = k_e \frac{|q|}{r}\hat{x} = (8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\frac{15 \times 10^6 C}{(0.20m)^2} = 3.4 \times 10^4 N/C$$

What is the force acting on a particle with charge $0.50\mu C$ at point P?

Bonus Round

Which way would the electric field point if q was negative?

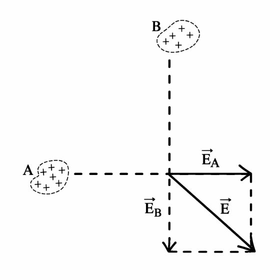


The Superposition of Electric Fields

At a particular point in space, each of the surrounding charges contributes to the net electric field that exists there. To determine the net field, it is necessary to obtain the various contributions separately and then find the vector sum of them all. Such an approach is an illustration of the principle of linear superposition, as applied to electric fields. The following example demonstrates this.

Example 4: The Superpostition of Electric Fields

Consider two charged objects, A and B, Each contributes as follows to the net electric field at point P at the coordinates (0,0). $\vec{E}_A = 3.00 N/C$ directed to the right, and $\vec{E}_B = 2.00 N/C$ directed downward. Thus, $\vec{E}_A$ and $\vec{E}_B$ are perpendicular. What is the net electric field at P?



The magnitude of the net electric field is

$$E = \sqrt{E_A^2 + E_B^2} = \sqrt{(3.00\,\mathrm{N/C})^2 + (2.00\,\mathrm{N/C})^2} = \boxed{3.61\,\mathrm{N/C}}$$

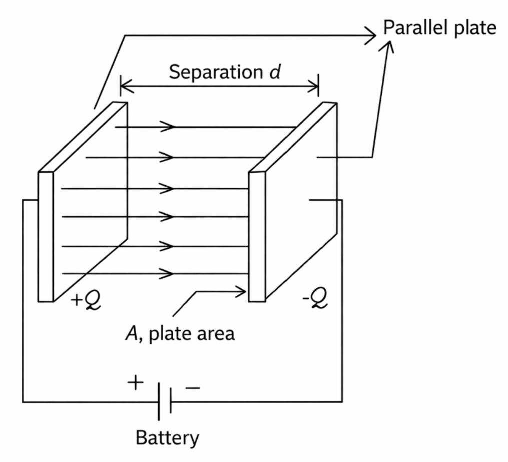
The direction of $\vec{E}$ is given by the angle $360^0 - \theta$ because net force in quadrant IV:

$$\theta = \tan^{-1}\left(\frac{E_B}{E_A}\right) = \tan^{-1}\left(\frac{2.00\,\mathrm{N/C}}{3.00\,\mathrm{N/C}}\right) = 33.7^\circ$$

So $360^0 - \theta = 326.3^0$ is the angle of the net electric field with respect to the positive x-axis.

The Parallel Plate Capacitor

A **capacitor** is a device used to store electric charge. The simplest type of capacitor is a **parallel plate capacitor**, which consists of two flat conducting plates placed near each other, separated by air or another insulating material

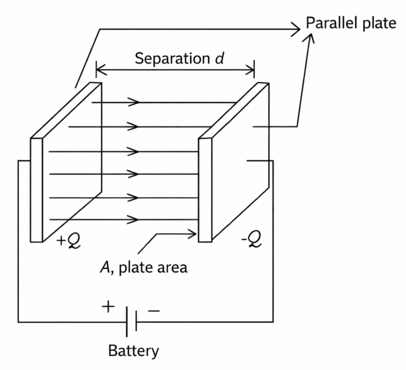


The Parallel Plate Capacitor

When charge is placed on a parallel plate capacitor, one plate becomes positively charged, and the other becomes negatively charged by the same amount. This creates a separation of charge, which results in an electric field between the plates.

The amount of charge a capacitor can store depends on:

- The **size of the plates** – larger plates can hold more charge.
- The **distance between the plates** – closer plates create a stronger attraction between opposite charges.



- The **material between the plates** – different materials can influence how much charge is stored.

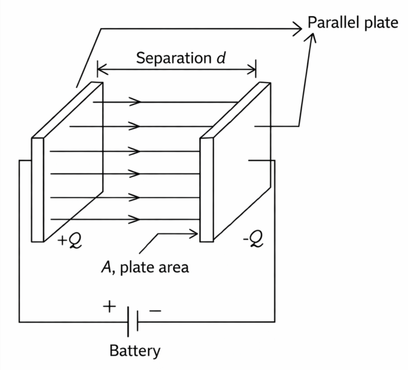
The Parallel Plate Capacitor

An **electric field** forms in the space between the plates.

$$E = \frac{q}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0}$$

where:

- q is the charge on one of the plates,
- A is the area of each plate,
- $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m is the permittivity of free space,
- $\sigma = \frac{q}{A}$ is the **surface charge density**, which represents charge per unit area on the plates.



This equation tells us that the electric field depends only on the charge per unit area and not on the separation distance (as long as the plates are close together and the field is uniform).

Electric Field Lines

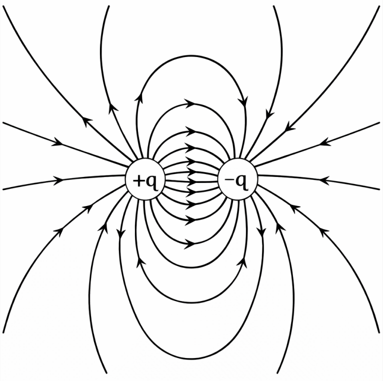


Figure 3: The electric field lines of an electric dipole are curved and extend from the positive to the negative charge. At any point, such as 1, 2, or 3, the field created by the dipole is tangent to the line through the point.

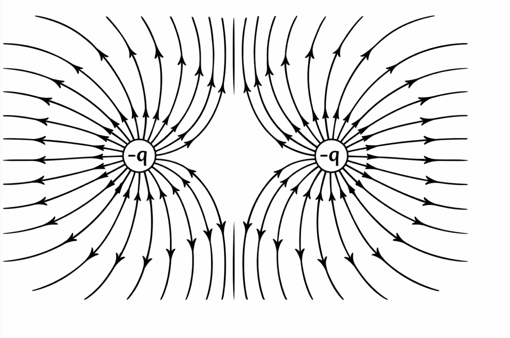


Figure 4: The electric field lines for two identical positive point charges. If the charges were both negative, the directions of the lines would be reversed.

The Electric Field Inside a Conductor

When a conductor is placed in an external electric field, free electrons inside the conductor move until the charges redistribute themselves in such a way that they cancel out any electric field within the material of the conductor.

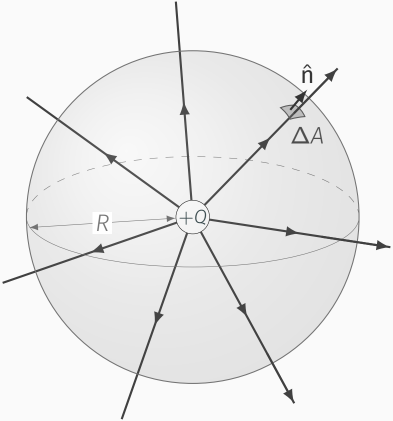
- Any **net charge** on a conductor resides on its **outer surface**.
- The **electric field inside the conductor is zero**.
- The conductor acts as a barrier, preventing external fields from influencing the space inside it.

This is known as **electrostatic shielding**. Example: A **Faraday cage** is a conducting enclosure that blocks external electric fields.

- **Car bodies**: If a lightning bolt strikes a car, the electric field inside remains zero, protecting passengers.
- **Microwave ovens**: The metal grid in the door prevents microwaves from escaping.
- **Shielded cables**: Coaxial cables use a conducting outer layer to prevent interference.

Gauss' Law

Consider a an imaginary surface around a point charge. This surface is called a **Gaussian Surface**.



Consider a small patch of this imaginary surface, ΔA. How much of the field goes through that patch?

$$\Delta\Phi_E = E\Delta A\cos\theta = E\Delta A$$

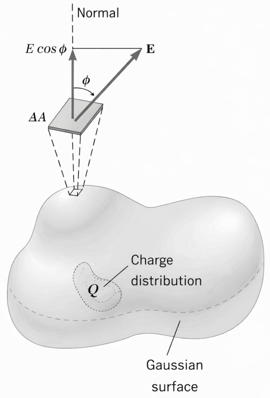
where θ is the angle between the electric field and the normal vector to the surface which for a sphere is 0^0. If we covered the sphere in such patches, and summed them up,

$$\Phi_E = \sum_i E\Delta A_i = E\sum_i \Delta A_i = EA$$

For a point charge, $E = q/4\pi r^2$, and the surface area of a sphere is $A = 4\pi r^2$, so $E = q/A\varepsilon_0$! So,

$$EA = \frac{q}{\varepsilon_0}$$

Gauss' Law



The electric flux, Φ_E, depends only on the charge, q, within the Gaussian surface and is independent of its radius! If we call that enclosed charge Q_{enc}, then we can write the electric flux as:

$$\Phi_E = \frac{Q_{enc}}{\varepsilon_0}$$

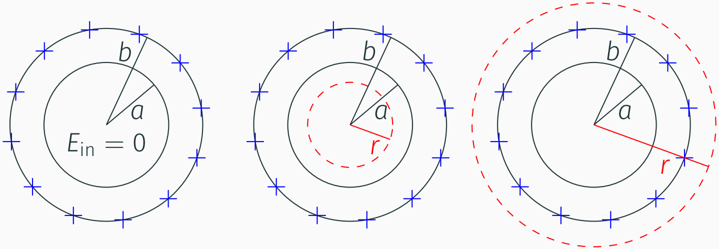
$$\lim_{N \to \infty} \sum_{i=1}^{\infty} E_i \Delta A_i \cos\phi_i = \frac{Q_{enc}}{\varepsilon_0}$$

And this holds for any closed surface, not just a sphere! This is **Gauss' Law**, one of the four Maxwell's equations that govern all of electromagnetism. When we can choose a Gaussian surface that has E with constant magnitude over its surface and is parallel or perpendicular to ΔA_i,

$$\boxed{EA = \frac{Q_{enc}}{\varepsilon_0}}$$

Example 6: The Electric Field of a Charged Spherical Shell

A spherical conducting shell of inner radius a and outer radius b carries a total charge $+Q$ distributed on the surface of a conducting shell. Find the electric field (a) in the interior of the conducting shell for $r < a$ and (b). outside the shell for $r > b$. (c) If an additional charge of $-2Q$ is placed at the center, find the electric field for $r > b$. (d). What is the distribution of charge on the sphere in part (c)?



(I) (II) (III)

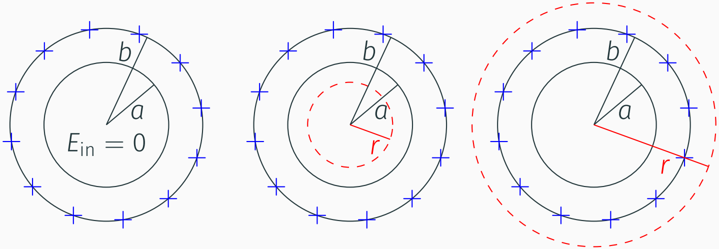
Example 6: The Electric Field of a Charged Spherical Shell

Solution:

(a). (Case II) $Q_{enc} = 0 \rightarrow E = 0$

(b). (Case III) $EA = E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$

(c). $EA = E(4\pi r^2) = \frac{Q_{inside}}{\varepsilon_0} = \frac{Q-2Q}{\varepsilon_0} \rightarrow E = -\frac{Q}{4\pi\varepsilon_0 r^2}$



(I) (II) (III)

Example 6: The Electric Field of a Charged Spherical Shell

Solution:
(d). Find the charge distribution on the sphere for part (c).

$$EA = E(4\pi r^2) = \frac{Q_{inside}}{\varepsilon_0} = \frac{Q_{center} + Q_{innersurface}}{\varepsilon_0}$$

The charge on the inner surface of the shell:

$$Q_{center} + Q_{innersurface} = 0$$

because the electric field in the conductor is zero. So

$$Q_{innersurface} = -Q_{center} = 2Q$$

Find the charge on the outer surface, noting that the inner and outer surface charges must sum to $+Q$:

$$Q_{outersurface} + Q_{innersurface} = Q$$

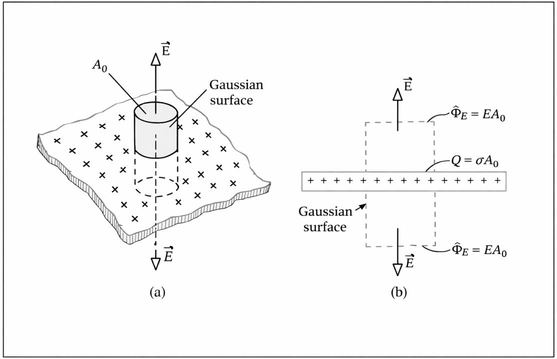
$$Q_{outersurface} = -Q_{innersurface} + Q = -Q$$

Example 7: A Nonconducting Plane Sheet of Charge

Find the electric field above and below a nonconducting infinite plane sheet of charge with uniform positive charge per unit area σ

Reasoning:

By symmetry the electric field must be perpendicular to the plane and directed away from it on either side as shown in (b). For the Gaussian surface, choose a small cylinder with axis perpendicular to the plane, each end having and area of A_o.



No electric field lines pass through the curved surface of the cylinder, only through the two ends, which have total area $2A_o$.

Example 7: A Nonconducting Plane Sheet of Charge

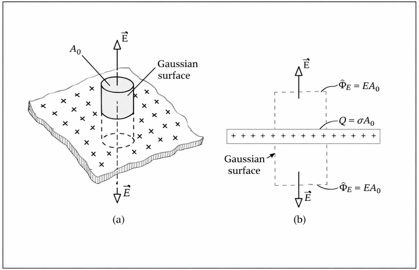
Find the electric field above and below a nonconducting infinite plane sheet of charge with uniform positive charge per unit area σ

Solution: Use Gauss's law:

$$EA = \frac{Q_{inside}}{\epsilon_o}$$

The total charge inside the Gaussian cylinder is the charge density times the cross-sectional area:

$$Q_{inside} = \sigma A_o$$



The electric flux comes entirely from the two ends, each having area A_o. Substitute $A = 2A_o$ and Q_{inside} and solve for E.

$$E = \frac{\sigma A_o}{(2A_o)\epsilon_o} = \frac{\sigma}{2\epsilon_o}$$

This is the magnitude of the electric field. We need the z-component of the field above and below the plane.

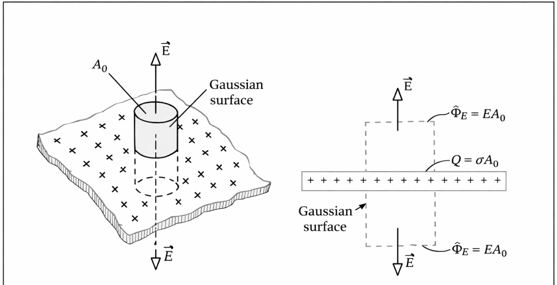
Example 7: A Nonconducting Plane Sheet of Charge

Find the electric field above and below a nonconducting infinite plane sheet of charge with uniform positive charge per unit area σ

Solution: (cont.)
The electric field points away from the plane, so it's positive above the plane and negative below the plane.

$$E_z == \frac{\sigma}{2\epsilon_0}, z > 0$$

$$E_z = -\frac{\sigma}{2\epsilon_0}, z < 0$$



(a) (b)

Electric Potential Energy

When we drop a basketball from some height h, gravity does work on the ball, converting its potential energy, $U_g = mgh$, into kinetic energy. Similarly, when we place a positive charge q in an electric field created by another charge Q, the electric field does work on the charge, converting its electric potential energy into kinetic energy. The electric potential energy U_e of a charge q in an electric field with potential V is given by:

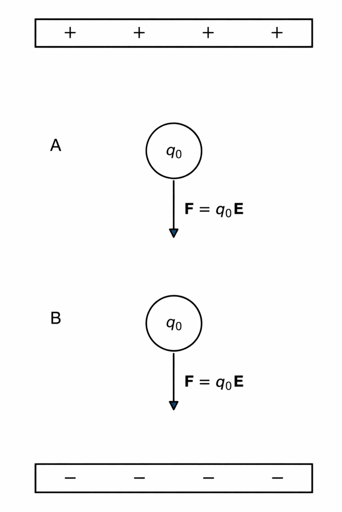
$$U_e = qV$$

where V is the electric potential at the location of the charge q due to other charges. The electric potential V is defined as the work done per unit charge by an external force in bringing a positive test charge from infinity to that point.

$$\text{gravitational potential} = gh \rightarrow \text{ analagous } \rightarrow V = \text{electric potential}$$

Just like gh is the gravitational potential energy per unit mass, V is the electric potential energy per unit charge.

Electric Potential Difference and Work



A positive test charge $+q_0$ placed inside and electric field, will experience an electric force $\mathbf{F} = q_0\mathbf{E}$. The force is doing work on the charge, changing its potential energy. The work done by the electric field in moving the charge from point A to point B is given by:

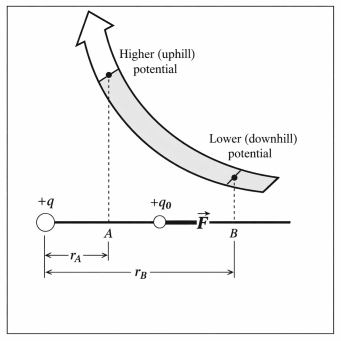
$$\begin{aligned} W_{AB} = -\Delta U_e &= -\left(U_{e,B} - U_{e,A}\right) \\ &= -\left(q_0 V_B - q_0 V_A\right) \\ &= -q_0(V_B - V_A) = -q_0\Delta V \end{aligned}$$

$$\Delta V = V_B - V_A = -\frac{W_{AB}}{q_0}$$

The minus sign indicates that the work done by the electric field reduces the potential energy of the charge.

The Electric Potential Difference Created by a Point Charge

Consider the electric potential difference created by a point charge $+q$ at a distance r.



For a point charge, the electric potential at a distance r from the charge is given by:

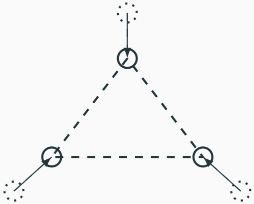
$$V = k_e \frac{q}{r}$$

which is easily derived in calculus. In the figure, a positive test charge $+q_0$ is moved from point A to point B in the electric field created by the point charge $+q$. The work done by the electric field:

$$W_{AB} = -q_0 \Delta V = -q_0 \left(k_e \frac{q}{r_B} - k_e \frac{q}{r_A} \right)$$

The work done by the electric field depends on the change in electric potential, which in turn depends on the distances r_A and r_B from the point charge.

Example: The Potential Energy of a Group of Charges



Three point charges are far apart and then brought together to form an equilateral triangle with sides of length $0.50\,m$. What is the total electric potential energy of this configuration of charges?

We calculate the electric potential energy by assembling the charges one at a time

Step 1: Place q_1 – No work is needed to bring in the first charge from infinity, since there are no other charges present:

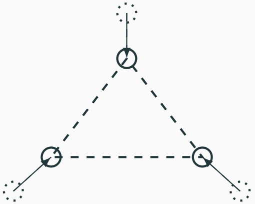
$$U_{e,1} = 0\ \mathrm{J}$$

Step 2: Place q_2 – The potential at the location of q_2 due to q_1 is:

$$V_1 = \frac{kq_1}{r} = \frac{(8.99 \times 10^9)(5.0 \times 10^{-6})}{0.50} = 9.0 \times 10^4\ \mathrm{V}$$

$$U_{e,2} = q_2 V_1 = (6.0 \times 10^{-6})(9.0 \times 10^4) = 0.54\ \mathrm{J}$$

Example: The Potential Energy of a Group of Charges



Three point charges are far apart and then brought together to form an equilateral triangle with sides of length 0.50 m. What is the total electric potential energy of this configuration of charges?

Step 3: Place q_3 – The potential at the location of q_3 is the sum of the potentials due to q_1 and q_2:

$$V_1 + V_2 = \frac{kq_1}{r} + \frac{kq_2}{r} = 2.0 \times 10^5 \text{ V}$$

Then the potential energy of q_3 is:

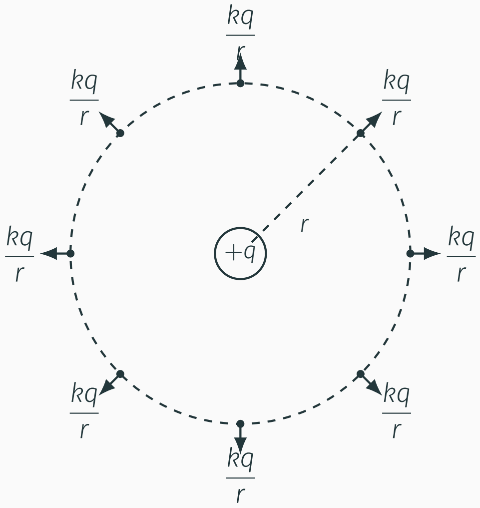
$$U_{e,3} = q_3(V_1 + V_2) = (-2.0 \times 10^{-6})(2.0 \times 10^5)$$
$$= -0.40 \text{ J}$$

Total Potential Energy of the System:

$$U_e = U_{e,1} + U_{e,2} + U_{e,3} = 0 + 0.54 - 0.40 = \boxed{+0.14 \text{ J}}$$

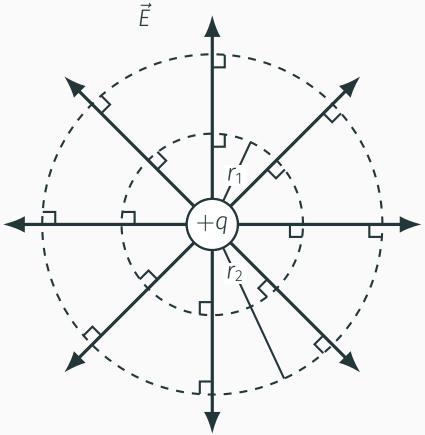
Equipotential Surfaces

An equipotential surface is just what its name declares: a surface - real or imaginary - on which all points have the same electric potential. Consider any point on a sphere around a point charge. All points have the same distance, r, so they all have the same potential $V = kq/r$. So how much work is needed to move a test charge, q_o along the surface of the sphere?



Equipotential Surfaces

Consider two of the equipotential spheres around a positive point charge. At each location on either of the equipotential spheres the electric field is perpendicular to the surface and points outward in the direction of decreasing potential. How do we know that the second equipotential surface is at a lower potential? Well $r_2 > r_1$ and the electric potential is inversely related to the distance so a bigger r results in a lower potential for the same charge.



Example: Work and Motion on Equipotential Surfaces

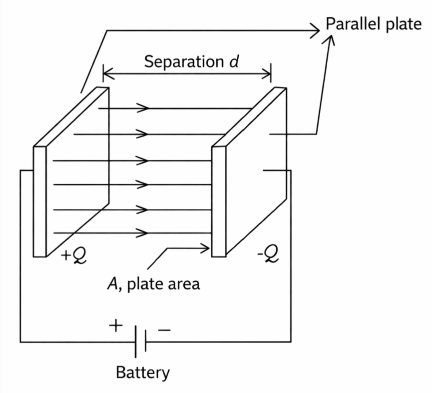
A positive test charge $q_0 = +1.0\ \mu\mathrm{C}$ is placed near a positive point charge $Q = +5.0\ \mu\mathrm{C}$. The test charge is moved from point A to point B, where: Point A lies on an equipotential surface where the potential is $V_A = 1200\ \mathrm{V}$ and Point B lies on an equipotential surface where the potential is $V_B = 800\ \mathrm{V}$.

(a) How much work is done by the electric field as the test charge moves from point A to point B?

(b) Would the test charge move spontaneously from A to B? Explain.

(c) What would the work be if the charge moved along a path that stayed entirely on the equipotential surface at $V = 1200\ \mathrm{V}$?

Capacitance of a Parallel Plate Capacitor



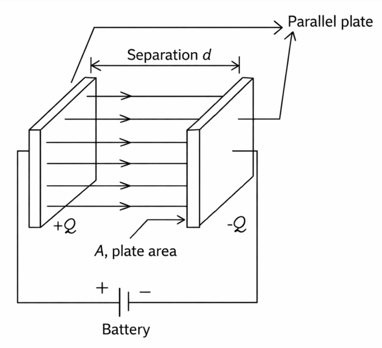
For a charge moving in **E**, $\Delta V = -W/q$. In that situation, potential decreases, so ΔV was negative because it was explicitly defined as $V_{final} - V_{initial}$.

For a capacitor we are not describing motion. We are comparing the potentials of two plates. Since the potential decreases in the direction of **E**, the plate from which the field originates must be at a higher potential.

$$V_{positive} - V_{negative} = \frac{W_{ext}}{q}$$

Capacitance relates the amount of charge stored to how much potential difference exists between teh plates – cannot be negative.

Capacitance of a Parallel Plate Capacitor



$$\Delta V = \frac{W_{ext}}{q}$$

$$W = F\cos\theta d = qE\cos\theta d = qEd$$

$$\Delta V = \frac{W}{q} = \frac{qEd}{q} = Ed \rightarrow E = \frac{\Delta V}{d}$$

E inside and empty parallel plate capacitor:

$$E = \frac{q}{\varepsilon_0 A}$$

$$\frac{\Delta V}{d} = \frac{q}{\varepsilon_0 A} \rightarrow \Delta V = \frac{qd}{\varepsilon_0 A}$$

Capacitance,

$$C = \frac{q}{\Delta V} = q \cdot \frac{\varepsilon_0 A}{qd} = \varepsilon_0 \frac{A}{d}$$

So for an **empty** parallel plate capacitor,

$$C = \varepsilon_0 \frac{A}{d}$$

Dielectrics

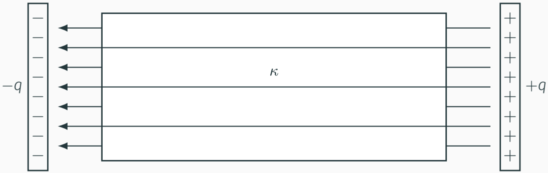
Recall that the electric field on the interior of a parallel plate capacitor is

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

Now lets stick some insulating material between the plates! This material is dialectric, meaning that it is made up of polarizeable molecules, which create their own electric field opposing the original field,

$$E = \frac{E_0}{\kappa} = \frac{\sigma}{\kappa\varepsilon_0}$$

where κ is the dieletric constant.



Repeating the stop from the previous slide,

$$\frac{\Delta V}{d} = \frac{\sigma}{\kappa\varepsilon_0}$$

$$C = \frac{q}{\Delta V} = q \cdot \frac{\kappa\varepsilon_0}{\sigma d} = q \cdot \frac{\kappa\varepsilon_0}{qd/A} = \kappa\varepsilon_0 \frac{A}{d}$$

$$C = \kappa\varepsilon_0 \frac{A}{d}$$

where $\kappa > 1$ for all dialectrics.

Dielectrics Constants

Substance	Dielectric Constant, κ
Vacuum	1
Air	1.000 54
Teflon	2.1
Benzene	2.28
Paper (royal gray)	3.3
Ruby mica	5.4
Neoprene rubber	6.7
Methyl alcohol	33.6
Water	80.4

Table 1: Dielectric constants κ for various substances.

Energy Stored in a Capacitor

The **battery does work** to move charge onto the plates *against* the electric field.

As charge builds up, the voltage increases:

$$V = \frac{Q}{C}$$

Since V rises linearly from 0 to V, the average voltage during charging is:

$$\bar{V} = \frac{1}{2}V$$

The total work done by the battery (stored as electric potential energy) is:

$$U = Q\bar{V} = \frac{1}{2}QV$$

Using $Q = CV$:

$$U = \frac{1}{2}CV^2 \qquad U = \frac{1}{2}\frac{Q^2}{C}$$